An Axiomatic Approach to Characterizing and Relaxing Strategyproofness of One-sided Matching Mechanisms [Extended Abstract]

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The *one-sided matching problem* is concerned with the allocation of indivisible goods to self-interested agents with privately known preferences. Monetary transfers are not permitted, which makes this problem different from auctions and other settings with transferable utility. In practice, such problems often arise in situations that are of great importance to peoples' lives. For example, students must be matched to schools, teachers to training programs, or houses to tenants. While strategyproofness is certainly a desirable design desideratum, it is also a severe restriction when other properties are important, such as efficiency or fairness. We study ordinal mechanisms for this problem, where agents have vNM utility functions over the objects. The paper makes two main contributions: we 1. characterize strategyproof one-sided matching mechanisms by three intuitive axioms, and we 2. propose the *partial strategyproofness* concept, a relaxation of strategyproofness, which bridges the long-standing gap between full and weak strategyproofness.

1. CHARACTERIZATION OF STRATEGYPROOF MECHANISMS

Our first contribution is to show that strategyproof mechanisms are characterized by three intuitive axioms. To understand the axioms, suppose an agent is considering whether to report truthfully or swap two adjacent objects, e.g., $a \succ b$ to $b \succ a$:

(1) A mechanism is *swap monotonic* if upon such a swap, the reporting agent's allocation either does not change at all, or the allocation for b strictly increases and the allocation for a strictly decreases.
(2) A mechanism is *upper invariant* if the allocation does not change for any object that the agent strictly prefers to a, i.e., any object in the upper contour set of a.
(3) The mechanism is *lower invariant* if the allocation does not change for any object that the agent likes strictly less than b, i.e., any object in the lower contour set of b.

We show that a mechanism is strategyproof *if and only if* it is swap monotonic, upper invariant, and lower invariant. Thus, strategyproofness requires that the mechanism only affects the allocation of the objects that are swapped (if any), and the direction of this change must be consistent with the agent's reported preferences.

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2. PARTIALLY STRATEGYPROOF MECHANISMS

Our second contribution is a new relaxation of strategyproofness, which we call *partial strategyproofness*. By dropping lower invariance, arguably the least intuitive axiom, the class of r-partially strategyproof mechanisms emerges. We show that these mechanisms are strategyproof for agents with sufficiently different values for different objects, i.e., for any two objects a, b with $a \succ b$ the agent's cardinal valuations must satisfy

$$r \cdot u(a) \geq u(b), \text{ assuming the normalization } \min u = 0. \tag{1}$$

To formally describe this relaxation of strategyproofness, we introduce the $\mathrm{URBI}(r)$ domain restriction: utilities for which inequality (1) holds satisfy *uniformly relatively bounded indifference* ($\mathrm{URBI}(r)$) with respect to $r \in [0,1]$. Geometrically, the utility functions from the shaded triangle in Figure 1 satisfy the $\mathrm{URBI}(r)$ constraint. A mechanism is *r-partially strategyproof* if it is strategyproof on the $\mathrm{URBI}(r)$ domain restriction. Our characterization result is that for a given setting $(N, M, \mathbf{q})$ (set of agents N, set of objects M, object capacities $\mathbf{q}$), a mechanism f is swap monotonic and upper invariant *if and only if* there exists $r \in (0,1]$ such that f is r-partially strategyproof in this setting.

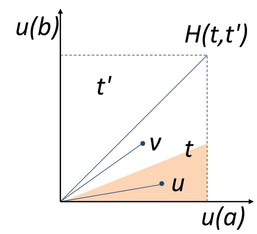

Fig. 1.

We further show that the $\mathrm{URBI}(r)$ domain restriction is maximal, i.e., $\mathrm{URBI}(r)$ is the largest set of utility functions for which partial strategyproofness can be guaranteed without knowledge of further properties of the mechanism beyond r-partial strategyproofness. This allows us to define a meaningful measure for the *degree of strategyproofness* of a non-strategyproof mechanism by considering the largest indifference bound $r \in [0,1]$ for which the mechanism is r-partially strategyproof:

$$\rho_{(N,M,\mathbf{q})}(f) = \max\left\{r \in [0,1] | f \text{ is } r\text{-partially strategyproof in } (N, M, \mathbf{q})\right\}. \tag{2}$$

Using ρ to compare the degree of strategyproofness of two mechanisms leads to results that are consistent with the method for comparing mechanisms by their vulnerability to manipulation proposed by Pathak and Sönmez [2013]. However, ρ has two advantages: it is *parametric*, and we show that it can be *computed algorithmically*.

Our new partial strategyproofness concept finds applications in the analysis of the incentive properties of existing mechanisms. First, while Random Serial Dictatorship is obviously r-partially strategyproof for any $r \in [0,1]$, we show that the non-strategyproof Probabilistic Serial is r-partially strategyproof for some $r > 0$. Second, the traditional "naïve" variant of the Boston mechanism is not r-partially strategyproof for any $r > 0$. However, we show that a small and natural adaptation yields a partially strategyproof variant [Mennle and Seuken 2014b]. Finally, partial strategyproofness can be used in the design of new hybrid mechanisms, which have interesting intermediate efficiency and strategyproofness properties [Mennle and Seuken 2014a]. We believe that the partial strategyproofness concept will lead to new insights in the analysis of existing non-strategyproof matching mechanisms and facilitate the design of new ones.

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