The Effect of Capital Taxation on Capital Accumulation and Welfare*

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Abstract

This paper analyzes the effects of time-consistent capital taxation on the level of capital and welfare. We find that a commitment to a zero capital tax shifts the time inconsistency problem towards labor taxes and the provision of public consumption. By comparing the worst time-consistent policies with and without a commitment to zero capital taxes, we find that the mere existence of a capital tax might lead to capital tax rates that are as high as 90% at steady state and capital stocks that are 84% lower. There the welfare gains of a commitment to zero capital taxes are about 7.4% of initial steady state consumption. At the other end, comparing the best time-consistent policies, we find that the welfare losses of a commitment to zero capital taxes are about 0.9% of consumption.

JEL Codes: E61, E62, H21, H62, H63.

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1 Introduction

As Kydland and Prescott (1977) and Fischer (1980) argue, the very existence of a capital income tax may lead to time-consistent capital tax rates that are too high and levels of savings that are too low. This paper explores this classic argument and quantifies the effects of capital taxes on the level of capital and welfare.

The literature on optimal taxation provides two central conclusions.[1] First, labor taxes should be roughly constant. Second, capital taxes should be zero in the long run and very high in the short run.[2] These conclusions have very different implications for time-inconsistency. While there seems to be little time-inconsistency in labor taxation, this problem becomes very severe in capital taxation. Chari et al. (1994) find that 80% of the welfare gains of switching from the current tax system to the Ramsey system comes from the high initial capital taxes. As the incentives to deviate from the announced zero capital taxes are paramount, some economists have suggested not to tax capital at all.[3,4]

This paper studies the effects of time-consistent capital income taxation on capital accumulation and welfare. More specifically, we consider a standard linear taxation problem. A benevolent government must finance an endogenous stream of public consumption through capital income taxes and labor income taxes.

We first consider the economy with commitment and analyze the effects of a mandated zero capital tax. The properties of optimal labor taxes without this constraint are well known. For homothetic utility functions, Chari and Kehoe (1998) show that labor taxes should be constant over time. Our results show that a mandated zero capital tax changes dramatically the properties of optimal labor taxes. The optimal labor tax rate is not longer constant but increasing over

[1] See Chamley (1986), Judd (1985), and Chari and Kehoe (1998).

[2] It is also well known, and recently illustrated by Conesa et al. (2009), that in life-cycle models and economies with borrowing constraints, the optimal capital tax is not zero in the long run. However, absent lump-sum taxation, the time-inconsistency problem of capital taxes is also present. It is then an open question the desirability of capital taxation without commitment in those setups.

[3] In one of his most influential works (see Lucas (1990)), Robert E. Lucas writes: "When I left graduate school, in 1963, I believed that the single most desirable change in the U.S. tax structure would be the taxation of capital gains as ordinary income. I now believe that neither capital gains nor any of the income from capital should be taxed at all."

[4] More recently, Mankiw et al. (2009) argue that capital income ought to be untaxed.

time.[5] In other words, governments find optimal to set current labor taxes low and announce higher labor taxes for the future. Therefore, we find that the time-inconsistency problem is now relocated towards labor taxes. Moreover, we find that a mandated zero capital tax leads to an underprovision of public consumption during the transition, which implies low welfare gains.

Next we study the economy without commitment. For economies with and without a mandated zero capital tax, we characterize the whole set of sustainable equilibria and, in particular, we focus on the properties of the best and the worst sustainable equilibria. We do this to quantify the effects of capital taxation on the level of capital and welfare in the best and worst possible scenarios. These extreme scenarios convey information to allow us to understand any equilibrium within the sustainable set.[6]

By comparing the worst time-consistent policies with and without a commitment to zero capital taxes, we find that the mere existence of a capital tax may lead to capital tax rates that are as high as 90 per cent and capital stocks that are 84 per cent lower. With capital taxes, the worst provides welfare losses in terms of initial steady state consumption of 28.2 per cent. A mandated capital tax reduces these losses to around 20.8 per cent. Therefore, in the worst possible scenario, the welfare gains of a commitment to zero capital taxes are about 7.4 per cent of consumption. Then, the effect of capital taxation on welfare is substantial but less dramatic than that on capital. This is so because with a commitment to zero capital taxes, the worst does not discourage savings but induces a substantially lower public consumption even at steady state.

At the other end, the best time-consistent policy with capital taxes provides welfare gains of 0.6 per cent of consumption. A mandated zero capital tax brings these gains down to -0.3 per cent. Therefore, at that end, we find that the welfare losses of a commitment to zero capital taxes are around 0.9 per cent. This comparison allows us to place the current US tax sytem as substantially close to the best (as opposed to the worst), but with potential gains from a better reputation. These gains, however, cannot be achieved through a mandated zero capital tax.

[5]The result that optimal labor taxes are increasing over time when there is a mandated zero capital tax is not that surprising after all. As Atkenson et al. (1999) observe, a positive tax on capital is comparable to an increasing tax rate on consumption and the latter corresponds to an increasing tax rate on labor income. These policies are comparable but not equivalent. In our economy with labor and capital taxes, a mandated zero capital tax eliminates an instrument that is necessary for decentralization and therefore changes the Ramsey problem.

[6]Moreover, a government cannot perfectly control the implementation of one sustainable equilibrium.

Since Kydland and Prescott (1977) uncovered the time-inconsistency problem of optimal policy plans, several papers have studied optimal capital taxation without commitment. Fischer (1980) is the first to illustrate this problem and its consequences. Chari and Kehoe (1990) use a model of intra-period capital accumulation to provide a game-theoretic formulation for the taxation problem. This formulation is extended to a dynamic setting by Phelan and Stacchetti (2001) and Sleet (1997). An alternative approach is used in Klein et al. (2008) to study optimal taxation without commitment. They focus on the properties of Markov-perfect equilibria, that is, when reputational mechanisms are absent. While the paper addresses a different question, their numerical results present a Markov-perfect equilibrium that seems quantitatively close to our worst sustainable equilibrium in terms of capital taxation and capital accumulation.

This paper is related to the work of Kydland and Prescott (1977), Lucas (1986) and Chari (1988). These papers already suggest the use of institutional changes to ameliorate time-inconsistency problems. Those institutional changes include implementation lags, monetary standards, budget balance, and the elimination of capital taxes among others.

There are a number of papers that explore the effect of institutional constraints on time-inconsistency. Among them, Athey et al. (2005) study the optimal degree of discretion in monetary policy and find that the best incentive-compatible equilibrium can be implemented by legislating an upper limit on inflation. Domínguez (2007a) shows that implementation lags in taxation together with the careful management of the maturity structure of government debt can make the optimal capital and labor tax plans time-consistent. Conesa and Domínguez (2012) examine social security as a commitment device. Domínguez (2010) studies the effect of debt limits on the time-inconsistency problems of default and devaluation of government debt.

Our results are related to those of Correia (1996) and Rogoff (1985). Correia (1996) studies capital taxation when another factor of production cannot be taxed and finds that the optimal steady state capital tax is not longer zero. In our model, the constraint that capital cannot be taxed changes the properties of optimal labor taxes. A similar result is also found in McCallum (1995)'s analysis of delegation in monetary policy as in Rogoff (1985). Delegation relocates the time-inconsistency problem to that of the choice of the conservative central banker. Our paper is also linked to the timeless perspective proposed by Woodford (2003). The timeless approach to optimal taxation requires governments not to take advantage of the economy's initial conditions. Our paper shows that a commitment to zero capital taxes is not sufficient to guarantee that initial conditions are not exploited.

The rest of the paper follows the following structure. Section 2 describes the model. Section 3 characterizes optimal labor taxes under a mandated zero capital tax when the government counts with commitment. Section 4 characterizes the economy without commitment and quantifies the effect of time-consistent capital taxes on capital accumulation. Section 5 concludes. Tables and details of the calibration and numerical algorithm are in the Appendix.

2 The Economy

The economy is populated by infinitely-lived identical individuals whose life-time utility is

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t, n_t) + G(g_t) \right], \tag{1}$$

with $\beta \in (0,1)$. The instantaneous utility $u(\cdot,\cdot) + G(\cdot)$ is a function of private consumption c_t, labor n_t and public consumption g_t, and takes the following form:

$$u(c_t, n_t) + G(g_t) = \frac{{c_t}^{1-\sigma}}{1-\sigma} - \gamma_n \frac{{n_t}^{1+\chi}}{1+\chi} + \gamma_g \frac{{g_t}^{1-\sigma}}{1-\sigma}, \tag{2}$$

where $\sigma \geq 0$ and $\chi \geq 0$ are respectively the inverse of the elasticity of intertemporal substitution of consumption and labor. The parameters $\gamma_n \geq 0$ and $\gamma_g \geq 0$ represent the weight on labor disutility and public consumption respectively. Individuals are endowed with initial capital k_0.

Taking prices and the government policy as given, the representative individual chooses consumption, labor and asset holdings to maximize his welfare (1) subject to the budget constraint[7]

$$R_t k_t + (1 - \tau_t^n) w_t n_t \geq k_{t+1} + c_t, \tag{3}$$

[7]We do not require private investment to be non-negative because governments cannot accumulate assets. In fact, in our numerical examples, private investment is always strictly positive.

and the no-Ponzi-game condition

$$\lim_{t\to\infty} p_t k_{t+1} \geq 0. \tag{4}$$

Here p_t is the price of the final good, w_t the real wage, τ_t^n the tax rate on labor income, R_t the gross return on capital, after tax τ_t^k and depreciation δ rates, and r_t the net return on capital, that is, $R_t = 1 + (1-\tau_t^k)(r_t - \delta)$, at date t. The first-order conditions for this problem are

$$-u_{n,t} = (1-\tau_t^n) w_t u_{c,t}, \tag{5}$$

$$u_{c,t} = \beta R_{t+1} u_{c,t+1}, \tag{6}$$

where u_c and u_n denote the marginal utility with respect to consumption and labor, respectively. Other derivatives follow a similar notation.

A representative competitive firm produces the final good using the technology $y_t = f(k_t, n_t)$, where f is increasing, concave and continuous differentiable. Taking factor prices as given, the firm chooses capital and labor to maximize profits, which implies

$$r_t = f_k(k_t, n_t) \text{ and } w_t = f_n(k_t, n_t). \tag{7}$$

We consider a benevolent government that must finance an endogenous public consumption g_t with taxes on labor income and on capital income. To reduce the dimensionality of the problem, we assume that the government has no access to public debt (and there is no initial debt). We later discuss how government bonds may affect the results. Then the government's budget constraint is

$$\tau_t^n w_t n_t + \tau_t^k (r_t - \delta) k_t = g_t. \tag{8}$$

We assume upper and lower bounds on the tax rates $\tau_t^i \in T^i = \left\{[\underline{\tau}^i, \overline{\tau}^i] | 0 \leq \underline{\tau}^i \leq \overline{\tau}^i < 1\right\}$ for $i = \{k, n\}$. A commitment to zero capital taxes corresponds to $\underline{\tau}^k = \overline{\tau}^k = 0$.

To complete the model, we write down the resource constraint as

$$f(k_t, n_t) + (1-\delta) k_t = c_t + k_{t+1} + g_t, \tag{9}$$

and define a competitive equilibrium in what follows:

Definition 1 *Given the policy* $\{\tau_t^k, \tau_t^n, g_t\}_{t=0}^{\infty}$*, and initial capital* k_0*,* $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ *is a competitive equilibrium allocation if and only if there exists a price sequence* $\{p_t, r_t, w_t\}_{t=0}^{\infty}$ *such that: (i) the representative individual maximizes welfare (1) subject to the budget constraint (3) and no-Ponzi game condition (4); (ii) factors are paid their marginal products (7); and (iii) all markets clear (the resource constraint (9) holds with equality).*[8]

3 Ramsey Taxes with Commitment

In this Section we study a Ramsey taxation problem. As the problem without a mandated zero capital tax is well understood,[9] here we focus on the problem with this constraint. Therefore we assume the following:

1. The government at date 0 is committed to a zero capital tax rate for all periods. This assumption takes the form of $\tau_0^k = 0$ in period 0 and

$$u_{c,t-1} = \beta(1 + r_t - \delta)u_{c,t}, \text{ in all periods } t \geq 1. \tag{10}$$

2. All future governments are committed to follow the announced sequence of taxes by the government at date 0. This assumption will be relaxed in the next Section.

We follow the primal approach to solve for the optimal fiscal policy. We substitute the first-order conditions (5)-(7) into the budget constraint (3) to obtain for each period the implementability condition

$$u_{c,t}c_t + u_{n,t}n_t + u_{c,t}k_{t+1} = \frac{1}{\beta}u_{c,t-1}k_t, \tag{11}$$

whose right-hand side is replaced by $u_{c,0}R_0k_0 = u_{c,0}\left(1 + f_{k,0} - \delta\right)k_0$ in period 0.

We now define the government's optimization problem. The government at date 0 chooses the sequences $\{c_t, n_t, g_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize the welfare of the representative individual (1) subject to

[8] Given that (3) and (9) hold, the government budget constraint (8) is also satisfied in a competitive equilibrium.
[9] See Stockman (2001) for the study of Ramsey capital taxes under a balanced budget.

the resource constraint (9), the zero capital tax constraint (10), and the implementability condition (11), given the initial condition k_0, and transversality condition

$$\lim_{t\to\infty} \beta^t u_{c,t} k_{t+1} = 0. \tag{12}$$

The bounds on labor tax rates $\tau_t^n \in T^n$ are assumed not to bind.[10]

The Lagragian for this optimization problem is

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t \left\{ u(c_t, n_t) + G\left(g_t\right) + \lambda_t \left[u_{c,t}c_t + u_{n,t}n_t\right] + \left(\lambda_t - \lambda_{t-1}\right) u_{c,t}k_{t+1} \right. \\
&+ \mu_t \left[f(k_t, n_t) + (1-\delta)k_t - c_t - k_{t+1} - g_t\right] \\
&\left. + \theta_{t+1} \left[\beta(1 + f_{k,t+1} - \delta)u_{c,t+1} - u_{c,t}\right] \right\} - \lambda_0 u_{c,0} R_0 a_0.
\end{aligned} \tag{13}$$

Note that $\lambda_{-1} = 0$ and $\theta_0 = 0$. The solution to this problem satisfies constraints (9)-(11), and the following first-order conditions for consumption, labor and capital for all $t \geq 1$:[11]

$$c_t^{-\sigma} \left\{ 1 + \lambda_t(1-\sigma) - \sigma\left(\lambda_t - \lambda_{t-1}\right) \frac{k_{t+1}}{c_t} + \sigma \left(\frac{\theta_{t-1}}{c_t} - \frac{\theta_t}{c_t} R_t \right) \right\} = \mu_t, \tag{14}$$

$$\gamma_n n_t^{\chi} \left(1 + \lambda_t(1+\chi)\right) - c_t^{-\sigma} \theta_t f_{kn,t} = f_{n,t}\mu_t, \tag{15}$$

$$\gamma_g g_t^{-\sigma} = \mu_t, \tag{16}$$

$$\mu_t = \beta\mu_{t+1}\left(1 + f_{k,t+1} - \delta\right) + c_{t+1}^{-\sigma}\left(\lambda_t - \lambda_{t-1}\right) + \beta c_{t+1}^{-\sigma}\theta_{t+1} f_{kk,t+1}, \tag{17}$$

where μ_t, θ_t and λ_t are the Lagrange multipliers on (9), (10) and (11), respectively.[12]

Let's define $\Psi_t \equiv \sigma\left(\frac{\theta_{t-1}}{c_t} - \frac{\theta_t}{c_t} R_t\right) + \theta_t \frac{f_{kn,t}}{f_{n,t}}$, $\Gamma_t = \sigma \frac{k_{t+1}}{c_t}$ and denote $\tilde{\lambda}_t$ the multiplier on (11) when there are no restrictions on capital tax rates (similarly for $\tilde{\Gamma}_t$). Combining (14)-(15) together with (5) and (7), we find:

Proposition 1 *The Ramsey tax plan for all $t \geq 1$ is characterized as follows:*

[10] For case (ii) in Proposition 1, we allow for capital taxation and assume that the bounds on capital tax rates do not bind. In our numerical exercise, the bounds for labor tax rates never bind. When capital taxes are available, the upper bound on capital tax rates binds for few periods.

[11] The first order conditions in period 0 are different due to the initial wealth.

[12] As usual in the literature, we assume that an optimal interior solution exists.

(i) With a zero capital tax rate constraint, the optimal labor tax rate is

$$\tau_t^n = \frac{\lambda_t(\sigma+\chi) + (\lambda_t - \lambda_{t-1})\,\Gamma_t - \Psi_t}{1+\lambda_t(1+\chi)}.$$

(ii) Without such a constraint, the optimal labor tax rate is

$$\tau_t^n = \frac{\tilde{\lambda}_t(\sigma+\chi) + \left(\tilde{\lambda}_t - \tilde{\lambda}_{t-1}\right)\tilde{\Gamma}_t}{1+\tilde{\lambda}_t(1+\chi)}.$$

Proof. See the Appendix. ■

Proposition 1 characterizes optimal labor taxes. First, because the government cannot issue bonds, the distortionary cost of taxation λ_t may not be constant over time. To isolate the effect of a mandated zero capital tax, let us focus on situations where the cost of distortionary taxation is constant over time $\lambda_t = \lambda_{t-1}$. Then, in the absence of any constraints on capital taxation, optimal labor taxes are constant over time. However, in the presence of a zero capital tax constraint, optimal labor taxes vary over time. The change over time in labor tax rates is captured by Ψ_t, which in our numerical simulations is positive and decreasing towards zero. Therefore, a mandated zero capital tax rate calls for an optimally increasing labor tax rate.

From conditions (5)-(6), we see that taxing future labor income more than current labor income produces an implicit capital tax. In other words, the government manipulates the timing of labor tax rates in an attempt to reproduce the missing tax instrument. Then, a mandated zero capital tax relocates the time-inconsistency problem towards that of labor taxes.

[Insert Table 1 and Figure 1 about here.]

To quantify the effect of a mandated zero capital tax, we compute the welfare gains (in terms of initial steady state consumption) of switching from the U.S. fiscal system to a Ramsey regime with and without capital taxes. For our baseline parameters with $\chi = 0.5$,[13] Table 1 shows that the welfare gains of a Ramsey reform without and with zero capital taxes are around 0.67 and 0.09 per cent of initial consumption, respectively. Therefore, the welfare gains of a Ramsey reform

[13]The details of the calibration are relegated to the Appendix.

without capital taxes are very small. Figure 1 provides the explanation for this. By eliminating the short-run high capital taxes, the government gives up a source of less distortionary taxation and, in turn, reduces public consumption during the transition.

In the spirit of Chari et al. (1994), we illustrate the severity of the time-inconsistency problem by computing the fraction of the welfare gains of the Ramsey regime with a mandated zero capital tax that comes from the timing of taxation and those that come from the change in taxation levels. The first (timing) indicates the future incentives to revise the policy. For our parameters, we obtain that most of these gains come from the timing of taxation, around 55 per cent, as opposed to the level, around 45 per cent.

As we later discuss, the welfare gains and their composition are sensitive to the parameters governing the utility from public consumption.[14] These results are also very sensitive to changes in the elasticity of labor supply. As illustrated in Table 1, we obtain that, as the elasticity of labor supply increases (lower χ), the welfare gains are lower and the proportion of those gains that comes from the timing of taxes is higher. The intuition is that a higher elasticity of labor supply increases the distortions of labor taxation bringing down the welfare gains and, at the same time, reduces the cost from not smoothing labor taxes facilitating changes in the timing of taxes to mimick the missing capital tax.

For values of χ in 0.32-0.50,[15] we see that in our economy with a mandated zero capital tax, the relatively high gains from the timing of taxation indicate that the time-inconsistency problem is severe, although the absolute low gains from the tax reform might indicate otherwise. Therefore it is not clear whether eliminating capital taxes might be enough to encourage levels of savings that are substantially higher and to reduce potential welfare losses from a bad reputation.

[14] To focus on taxation rather than level of public consumption, we calibrate γ_g so that the government spending to output ratio of the social planner's solution coincides with our benchmark economy and set the elasticity of intertemporal substitution of public consumption to the same level as that of private consumption.

[15] Rogerson and Wallenius (2009) suggest elasticities of labor supply in the range of 2.25-3.00, which in our economy coincide with values of χ in 0.33-0.44.

4 Ramsey Taxes without Commitment

In this Section we compute the time-consistent optimal fiscal policy with and without a commitment to zero capital tax rates. That is, we allow future governments to reconsider the policy.

To facilitate the computation of the equilibrium set, we make some changes to the model. First, the instantaneous utility is normalized by $(1-\beta)$. Second, we introduce a public randomization device to convexify the payoff set of the government at a given value of (k, m), which we interpret as the government playing a mixed strategy, and to synchronize the government's and the households' moves and beliefs. Finally, we assume that individuals are anonymous. Then the actions of an individual are not observed neither by the government nor by other individuals, ruling out any possible coordination of actions. Moreover, given identical beliefs, the convexity of the individual's problem ensures that all choose the same actions. Then, our analysis concentrates on symmetric competitive equilibria.

As Phelan and Stacchetti (2001) show, the individual's problem can be written recursively by including as a state variable the marginal value of capital m_{t+1},[16] i.e.

$$m_{t+1} \equiv u_{c,t+1}[1 + (1 - \tau^k_{t+1})(r_{t+1} - \delta)]. \tag{18}$$

Using the recursive formulation and denoting next period variables with a subscript $+$, we define a competitive equilibrium as follows:

Definition 2 *The vector* (c, n, k_+, g, w, r) *constitutes a competitive equilibrium, denoted* $(c, n, k_+, g, w, r) \in$

[16]For the utility function (2), Feng (2011) shows that the sequential and recursive problems are equivalent.

$CE(k, \tau^k, \tau^n, m_+)$, if and only if

$$u_c(c,n) = \beta m_+, \tag{19}$$

$$-u_n = (1-\tau^n) w u_c, \tag{20}$$

$$k_+ = [1 + (1-\tau^k)(r-\delta)]k + (1-\tau^n)wn - c, \tag{21}$$

$$g = \tau^n wn + \tau^k (r-\delta)k, \tag{22}$$

$$w = f_n(k,n), \tag{23}$$

$$r = f_k(k,n). \tag{24}$$

We denote the public history by $\zeta^t = (\zeta_1, ..., \zeta_t)$, where $\zeta_t = (\tau_t^n, \tau_t^k, k_{t-1}, r_t, w_t)$, that is, the history of government policies, capital stock, and aggregate prices. In our sequential equilibria, the government chooses first. A strategy for the government at date t, denoted $\sigma_{G,t}(\zeta^{t-1})$, is a choice of current taxes as a function of the history ζ^{t-1}. Households choose second. A symmetric strategy for them at date t, denoted $\sigma_{H,t}(\zeta^t)$, is a choice of a current allocation as a function of the public history ζ^t. After each history ζ^{t-1}, a strategy profile (σ_H, σ_G) induces a continuation strategy profile. A strategy profile induces an outcome, which produces a payoff for the government and a payoff for the households. This allows us to define a sustainable equilibrium.

Definition 3 *A symmetric strategy profile (σ_H, σ_G) is a sustainable equilibrium if it satisfies the following conditions for all $t \geq 0$:*
(i) given the symmetric strategy for households $\sigma_{H,t}$, the continuation payoff for the government is higher than the payoff from any deviation to a different strategy $\sigma'_{G,t}$ for every history ζ^{t-1}; and
(ii) given the strategy for the government $\sigma_{G,t}$, the continuation payoff for the household is higher than the payoff from any deviation to a different strategy $\sigma'_{H,t}$ for every history ζ^t.

The definition of sustainable equilibrium we apply is the same as the one in Chari and Kehoe (1990) and the one of symmetric sequential equilibrium in Phelan and Stacchetti (2001). This definition builds on two conditions that guarantee sequential rationality. The first requires the government not to have incentives to deviate and the second requires individuals to behave competitively.

Our basic framework is similar to the one used by Phelan and Stacchetti (2001). They show how to write our dynamic policy game recursively and apply the APS method (Abreu et al. (1990)). Basically, one requires a continuation value for households, denoted m_+, and a continuation value for the government, denoted h_+, to define the set of values (m, h) that can be attained in a sustainable equilibrium. For a given initial capital k, this set of values is called the equilibrium value correspondence $\mathbf{V}(k)$. In what follows, we detail the numerical algorithm that we use to compute this value correspondence.

4.1 Computation of equilibria

We first define an arbitrary value correspondence $\mathbf{W}$ as any mapping from k into sets of payoffs (m, h). Then we define consistency:

Definition 4 *The vector $\psi = (\tau^k, \tau^n, c, n, k_+, g, w, r, m_+, h_+)$ is said to be consistent with respect to the value correspondence $\mathbf{W}$ at k if $(c, n, k_+, g, w, r) \in CE(k, \tau^k, \tau^n, m_+)$, $\tau_t^i \in T^i$, for $i = \{k, n\}$, $(m(k, \psi), h(k, \psi)) \in \mathbf{W}(k)$, and $(m_+, h_+) \in \mathbf{W}(k_+)$, where m is the marginal value of capital and h is the value of the vector ψ*

$$m(k, \psi) := u_c(c, n)[1 + (1 - \tau^k)(r - \delta)], \tag{25}$$

$$h(k, \psi) := u(c, n) + G(g) + \beta h_+. \tag{26}$$

Next we define admissibility as

Definition 5 *The vector $\psi = (\tau^k, \tau^n, c, n, k_+, g, w, r, m_+, h_+)$ is said to be admissible with respect to the value correspondence $\mathbf{W}$ at k if it is consistent and*

$$h(k, \psi) \geq \left[u(c, n) + G(g) + \beta h_+^{'} \mid \left(m_+^{'}, h_+^{'}\right)\right] \quad \forall \left(m_+^{'}, h_+^{'}\right) \in \mathbf{W}(k_+). \tag{27}$$

Note that admissibility captures the two conditions required in the definition of a sustainable equilibrium. Through consistency, it satisfies that individuals behave competitively. Through constraint (27), which is the incentive compatibility constraint, it implies that the government does not want to deviate.

We then define an operator $\mathbb{B}$, $\mathbb{B} : \mathbf{A} \to \mathbf{A}$, where $\mathbf{A}$ is the space of all value correspondences. The operator $\mathbb{B}$ is the convex hull of all sets (m, h) that satisfy admissibility (and therefore consistency). That is, the payoffs (m, h) that form part of a sustainable equilibrium. Computing the mapping $\mathbb{B}$ amounts to find a set $\mathbb{B}(\mathbf{W})$, that is the set of (m, h) that can be "enforced" today:

$$\mathbb{B}(\mathbf{W})(k) = \left\{(m,h)|\exists(\tau^k, \tau^n, c, n, k_+, g, w, r, m_+, h_+) \text{ that are admissible w.r.t. } \mathbf{W}\text{at } k\right\}.$$

As pointed out by Chang (1998), computing $\mathbb{B}(\mathbf{W})$ given $\mathbf{W}$ is complicated in particular by the presence of constraint (27). Chang (1998) suggests an alternative operator to circumvent this complication in the context of finding time-consistent monetary policies, while Phelan and Stacchetti (2001) develop a similar operator in a production economy like ours.

The basic idea of a simpler approach is the following. On the one hand, the government does not need to evaluate the consequences of all possible actions, it only needs to consider the payoff associated with the "best" deviation. On the other hand, and as in Abreu et al. (1990), if the government chooses to deviate, this is then followed by the worst available punishment. Hence, we replace the condition (27) with the following one:

$$h(k, \psi) \geq \tilde{h}(k), \tag{28}$$

where $\tilde{h}(k)$ is the worst possible payoff for the government when it deviates. As mentioned above, we only consider extreme punishments. Therefore, we define $\tilde{h}(k)$ as

$$\tilde{h}(k) = \max_{\tau^k, \tau^n} \left\{ \min_{c,n,k_+,(m_+,h_+)\in\mathbf{W}(k_+)} \left[u(c,n) + G(g) + \beta h_+\right] \right\},$$

such that $(c, n, k_+, g, w, r) \in CE\left(k, \tau^k, \tau^n, m_+\right)$.

To facilitate the computation of $\mathbb{B}(\mathbf{W})$, Phelan and Stacchetti (2001) introduce a public randomization device to convexify the equilibrium set so that the innovative approximation technique developed by Judd, Yeltekin and Conklin (2003) can be applicable. We also introduce a public randomization to make sure that $\mathbf{W}$ is convex-valued, but instead we apply the numerical method developed by Feng et al. (2011) to approximate equilibrium sets, which can be used to approximate convex-valued sets and has a good convergence property as stated in Theorem 4.1 in their paper.

We should emphasize that we compute the upper and lower boundaries of $\mathbf{W}(k)$, which, as in Phelan and Stacchetti (2001), are represented by the following functions:

$$\bar{h}(k,m) := \max_h \left\{h|(m,h) \in \mathbf{W}(k)\right\}, \tag{29}$$

$$\underline{h}(k,m) := \min_h \left\{h|(m,h) \in \mathbf{W}(k)\right\}. \tag{30}$$

As observed by Phelan and Stacchetti (2001), the lowest value in $\mathbf{W}(k)$ yields the value of the worst punishment for the government $\tilde{h}(k) = \min_m \underline{h}(k,m)$, which corresponds to the equilibrium at the trigger strategy. The highest value $\max_m \bar{h}(k,m)$ in $\mathbf{W}(k)$ corresponds to the equilibrium that the government obtains the maximum payoff (at the best strategy). We are interested in the boundaries since we learn a lot about the equilibrium set from looking at the extremes. The following proposition will be useful for the computation of the boundaries:

Proposition 2 *If* $\mathbf{W}$ *is convex-valued at given* $\{k,m\}$*, and at given* k *and* $\left\{\left(\tau^k,\tau^n\right),m\right\}$*, there exists a vector* (c,n,k_+) *that solves*[17]

$$m = u_c\left[1+(1-\tau^k)(r-\delta)\right], \tag{31}$$

$$u_n + (1-\tau^n)wu_c = 0, \tag{32}$$

$$\left[1+(1-\tau^k)(r-\delta)\right]k + (1-\tau^n)wn - c - k_+ = 0, \tag{33}$$

then, for $(m,h) \in \mathbf{W}(k)$*, we obtain that*

$$\bar{h}(k,m) = \max_{\tau^k,\tau^n} u(c,n) + G(g) + \beta\bar{h}(k_+,m_+), \tag{34}$$

$$\underline{h}(k,m) = \max\left\{\max_{\tau^k,\tau^n} u(c,n) + G(g) + \beta\underline{h}(k_+,m_+), \tilde{h}(k)\right\}. \tag{35}$$

[17]This system can be simplified into one non-linear equation in terms of n for the assumed utility function (2).

Proof. See the Appendix. ■

In the above Proposition, we see that, while the continuation value of a best is also a best, the continuation value of the worst might not be the worst. That might happen when the incentive constraint is violated at a given k. This proposition allows us to concentrate on the boundaries of the value correspondence. This will greatly contain the computational cost. To this end, we find an outer approximation of $\mathbf{W}$: $\hat{\mathbf{W}}(k) = \left\{(m,h)\,|h \in \left[\underline{h}(k,m), \bar{h}(k,m)\right]\right\}$.

In the Appendix, we define an operator $\mathbb{F}$ and briefly explain our algorithm. We refer to Feng et al. (2011) for details in the approximation of convex-valued sets.

4.2 Results

In this Subsection, we present the results from the numerical approximation of the value correspondence for two distinct setups: economy A (without a commitment to zero capital taxes) and economy B (with a commitment to zero capital taxes). Both economies share the same initial capital stock and parameters. Table 2 summarizes the steady state characterization of the best and worst sustainable equilibria for economies A and B. Table 3 shows the welfare gains/losses (in terms of percentage change in initial steady state consumption) relative to our benchmark economy. Our benchmark, described in the Appendix, is an economy with a similar fiscal system and relevant statistics as those of the US economy.

[Insert Tables 2 and 3 about here.]

First, we compare the worst time-consistent policies with and without a commitment to zero capital taxes. That is, the worst possible scenario in terms of reputation that an economy may encounter. For economy A, Table 2 shows that steady state capital taxes are consistenly very high, around 90 per cent, and steady state capital stocks are very low, around one fifth of the values obtained at the best time-consistent equilibrium. For economy B, with no capital taxes, we see that steady state capital stocks are substantially larger. In fact, the steady state capital stock in economy A is 84 per cent lower than in economy B.

When governments are allowed to tax capital, Table 3 shows that the welfare losses are equivalent to 28.2 per cent of initial steady state consumption. When governments cannot tax capital, these losses go down to 20.8 per cent. Therefore, in the worst possible scenario, the welfare gains of a commitment to zero capital taxes are about 7.4 per cent of consumption. As can be seen, the differences in welfare are substantial but less dramatic that those for capital acummulation. The reason is that, under a commitment to zero capital taxes, worst beliefs induce a substantially lower provision of public consumption not only during the transition but also at steady state.

Next, we compare the best time-consistent policies with and without a commitment to zero capital taxes. This represents the other end, the best possible scenario in terms of reputation. For both economies, A and B, the steady state level of capital taxes is either close to zero or zero and the stock of capital is comparable in size. When governments can tax capital, the welfare gains of the best time-consistent policy are 0.6 per cent of consumption. When capital cannot be taxed, these gains go down to about -0.3 per cent. Therefore, at that end, we find that the welfare losses of a commitment to zero capital taxes are around 0.9 per cent of consumption.

Our sustainable equilibria are not recursive in k, neither imposed to be recursive in k (as it is the Markov-perfect). To make them recursive, the continuation values (m_+, h_+) have been added as new state variables, which summarize the public's expectations and the government's incentives. In our economy, a good reputation can almost substitute for commitment. The values of the BSE (best sustainable equilibrium) are close to those of the Ramsey (around a loss of 0.1 % in A and 0.4% of consumption in B). The allocation and policy are also similar except for positive capital taxes at the BSE as opposed to zero in A and a lower provision of public consumption in B. This shows that the incentive compatibility constraint $h(k, \psi) \geq \tilde{h}(k)$ binds only slightly in A and moderately in B. By looking at Table 2, given best beliefs, a government may provide good incentives and strategically influence the next by today taxing capital little and providing high public consumption.

In our economy, a bad reputation leads to large welfare losses. The values of the WSE (worst sustainable equilibrium) are very far from the Ramsey (around a loss of 29 % in A and a loss of 21 % in B). By looking at Table 2, given worst beliefs, a government in economy A ends up taxing capital heavily (even at steady state), depleting the capital stock and reducing the possibilities of consumption. In economy B, given worst beliefs, the capital stock is still large. However, worst expectations induce the government to spend too little in public consumption. Relating this to the results of Section 3, we find that the implicit capital tax through the timing of labor taxes is not enough to discourage savings and that the big reputational cost comes from a permanent underprovision of public consumption.[18]

As mentioned by Martin (2009), depending on the calibration, a Markov-perfect equilibrium may coincide with the worst-sustainable equilibrium. In our exercise, we did not compute the Markov-perfect equilibrium. However, comparing our (re-calibrated) worst with the Markov-perfect of Klein et al. (2008), ours displays a similar effect on capital accumulation but a much lower public consumption to output ratio.[19]

4.3 Limitations of the Results

There are a number of assumptions that limit our results. First, from Domínguez (2007a) and Reis (2011), we know that allowing governments to issue debt affects the properties of capital taxation without commitment. In our setup, one might expect that allowing for government debt would enlarge the equilibrium set of both economies A and B. The BSE would be higher and the WSE could be lower (particularly when default on debt has a negative effect on productivity) for both economies.

In terms of differences between economy A and B, at the BSE, it seems to us that government debt could benefit economy A relatively more. As the government would be able to accumulate more assets at the beginning (when capital taxation is less distortionary). At the WSE, government debt could damage economy B relatively less. Through varying labor taxes, economy B has an imperfect implicit capital tax and might display larger debt ratios for a given k increasing the relative gains from default. This makes us regard the above results, particularly those for the worst, as providing a lower bound for the effects of time-consistent capital taxation on capital accumulation and welfare.

[18]This effect is similar to the one found by Domínguez (2010) in an economy with government debt.

[19]Klein et a. (2008) consider a higher weight on public consumption,$\gamma_g = 0.49$, and find a public consumption to output ratio of 19 % in their Markov with only labor taxation. For this parameter, our BSE with a commitment to zero capital taxes provides welfare gains close to 2 % of initial consumption and a public consumption to output ratio of 24 %. At the WSE, the public consumption to output ratio is 8 %.

Another limitation of our results may come through the response of capital to capital taxation. The effect of capital taxation on capital accumulation clearly depends on how easy is to distinguish between capital and labor income. If individuals can convert capital income into labor income, the response of capital accumulation to high capital taxes may be very moderate (also the incentives to tax capital heavily).

Finally, and linked to the previous comment, our setup does not contemplate any of the potential reasons, such as uninsurable risk or a life-cycle structure, that render the Chamley-Judd result invalid and make a non-zero capital tax optimal even in the presence of commitment.[20] In those situations, our results may overstate any potential welfare gains of a commitment to zero capital taxes.

5 Conclusions

We have investigated the effects of capital taxation on capital accumulation and welfare. In the economy with commitment, we have shown that a mandated zero capital tax changes the properties of optimal labor taxes. Optimal labor taxes are now increasing over time. Moreover, there is an underprovision of public consumption during the transition. This implies a relocation of the time-inconsistency problem towards that of labor taxation and public consumption and suggests that eliminating capital taxes might not be enough to encourage capital accumulation and to reduce the welfare losses due to a bad reputation.

In the economy without commitment, we found that time-consistent capital taxes can be as high as 90 % and capital stocks can be 84% lower (compared to without no capital taxes). The welfare gains of a mandated zero capital tax are equivalent to 7.4 % at the worst and the losses are around 0.9 % at the best. While a commitment to zero capital taxes is able to encourage capital accumulation, worst beliefs induce a large and permanent underprovision of public consumption.

The current US tax sytem is relatively close to the best (as opposed to the worst), but there can be substantial gains from a better reputation. Those gains, however, cannot be achieved with a mandated zero capital tax. Such a constraint leads to an underprovision of public consumption

[20] See Conesa et al. (2009) for an overview of the literature.

and potentially large welfare losses. Our results predict that economies with bad reputations have very high capital taxes and extremelly low capital stocks or, alternatively, high capital stocks but lower public consumption ratios.

One important implication of this paper is that institutional arrangements need to be carefully designed and require coordination of policy instruments.

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6 Appendix

6.1 Figures and Tables

Figure 1: Public Consumption to Output Ratio

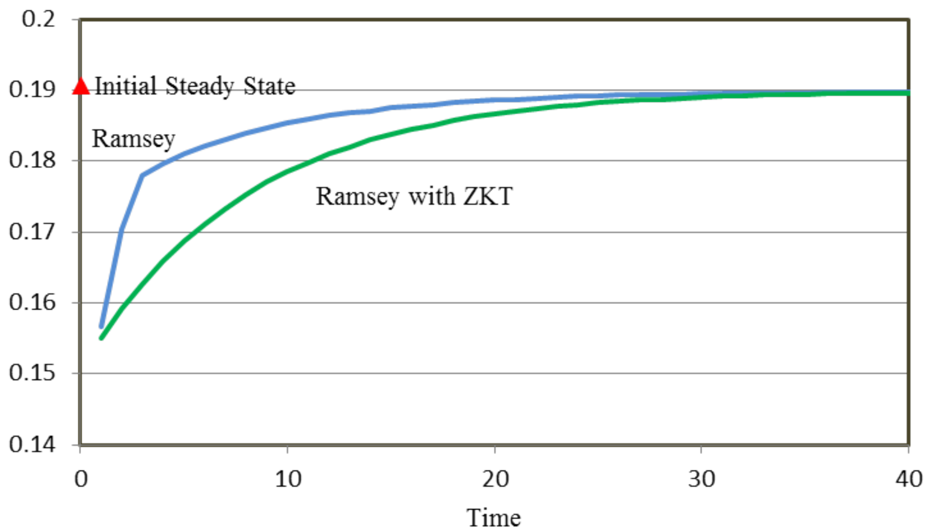

Figure 1: Public Consumption to Output Ratio

Table 1: Welfare Gains of a Ramsey Reform without and with ZKT and Decomposition

χ	Welfare Gains of Ramsey	Welfare Gains of Ramsey with ZKT (*)	Fraction of (*) from Time-Variant Labor Tax Rates	Fraction of (*) from Constant Labor Tax Rates
0.32	0.755	0.066	0.96	0.04
0.5	0.666	0.091	0.55	0.45
1.0	0.533	0.146	0.21	0.79
1.5	0.470	0.186	0.11	0.89

Note: ZKT stands for zero capital taxes.

Note: Welfare gains are in terms of % change in initial steady state private consumption.

Table 2: Final Steady State Allocation and Policy

	χ	Economy A $\bar{\tau}^k$	Economy A $\bar{\tau}^n$	Economy A $\frac{\bar{k}}{y}$	Economy A $\frac{\bar{g}}{y}$	Economy B $\bar{\tau}^k$	Economy B $\bar{\tau}^n$	Economy B $\frac{\bar{k}}{y}$	Economy B $\frac{\bar{g}}{y}$
Best Sustainable Equilibrium	0.32	0.122	0.262	2.900	0.188	0.00	0.285	3.006	0.188
	0.50	0.079	0.270	2.932	0.187	0.00	0.285	3.005	0.188
	1.00	0.067	0.270	2.940	0.190	0.00	0.286	3.005	0.189
	1.50	0.042	0.275	2.937	0.194	0.00	0.287	3.005	0.189
Worst Sustainable Equilibrium	0.32	0.900	0.001	0.860	0.250	0.00	0.025	3.004	0.016
	0.50	0.895	0.001	0.830	0.243	0.00	0.041	3.005	0.025
	1.00	0.895	0.001	0.829	0.250	0.00	0.059	3.005	0.039
	1.50	0.895	0.001	0.829	0.250	0.00	0.065	3.006	0.043

Table 3: Welfare gains/losses relative to the initial steady state

	χ	Economy A	Economy B
Best Sustainable Equilibrium	0.32	0.62	-0.48
	0.50	0.60	-0.30
	1.00	0.46	-0.24
	1.50	0.37	-0.03
Worst Sustainable Equilibrium	0.32	-31.05	-20.53
	0.50	-28.18	-20.78
	1.00	-24.83	-20.98
	1.50	-21.36	-19.77

Note: Welfare gains are in terms of % change in initial steady state private consumption.

6.2 Calibration

We start with a benchmark economy: a calibration of an initial steady state that corresponds to an economy with similar policy and statistics to those of the US economy. This initial steady state provides initial conditions for our Ramsey reform.

Our calibration relies substantially on that of Chari et al. (1994), which is consistent with U.S. data. We consider the utility function (2) and a Cobb-Douglas production function $y_t = Ak_t^{\alpha} n_t^{1-\alpha}$. In our simulations one period corresponds to one year. We assume a capital share in production of 0.34 and a depreciation rate of 0.08. The discount factor is chosen to obtain a capital to output ratio of 2.71 in the initial steady state. In the utility function the degree of relative risk aversion σ is set equal to unity and the labor-supply elasticity is set to 2 (χ= 0.5). The weight on labor is chosen so that hours worked is 0.23 in the initial steady state. The weight on public consumption γ_g is chosen so that the government spending to output ratio in the social planner's solution coincides with the one of our initial steady state, which is close to 19 per cent.

Policy parameters are as follows. There is no government debt, neither initial debt. The initial tax rates on capital and labor income are fixed to 27.1 and 23.7 per cent respectively. In the Ramsey with and without commitment, upper and lower bounds on tax rates are defined by $\tau^k \in [0.0, 0.0]$ and $\tau^n \in [0.0, 1.0]$ when there is a commitment to zero capital taxes, and $\tau^k, \tau^n \in [0.0, 1.0]$ otherwise. Table 4 shows our calibration targets and the parameters used to meet these targets. Table 5 summarizes the parameter values used in the baseline model and initial steady state.

Table 4: Targets in the Initial Steady State

β	Target: $\frac{K_{ss}}{Y_{ss}} = 2.71$
γ_n	Target: $n_{ss} = 0.23$
γ_g	Target: $\frac{G_{planner}}{Y_{planner}} = \frac{Gss}{Y_{ss}}$

Table 5: Parameter values for the baseline economy

Preference	$\beta = 0.968$	$\sigma = 1.0$	$\gamma_n = 7.694$	$\chi = 0.5$	$\gamma_g = 0.333$
Technology	$A = 1.0$	$\alpha = 0.34$	$\delta = 0.08$		
Policy	$\tau_0^n = 0.237$	$\tau_0^k = 0.271$			

To perform the Ramsey reform with and without commitment, we assume the same parameter values and set as initial conditions those resulting from the initial steady state. We find the Ramsey allocation with commitment by solving the optimization problem using a successive quadratic programming method. We obtain the Ramsey values and allocation without commitment by implementing the numerical algorithm explained below.

6.3 Numerical Implementation

For the numerical implementation of the algorithm we define an operator $\mathbb{F}$ as follows:

Definition 6 *For any convex-valued set* $\hat{\mathbf{W}} = \left\{(m,h)|h \in \left[\underline{h}^0(k,m), \bar{h}^0(k,m)\right]\right\}$*, we define operator* $\mathbb{F}$ *as follows:*

$$\mathbb{F}(\hat{\mathbf{W}})(k) = \left\{(m,h)|h \in \left[\underline{h}^1(k,m), \bar{h}^1(k,m)\right]\right\},$$

where

$$\bar{h}^1(k,m) = \max_{\tau^k,\tau^n} u(c,n) + G(g) + \beta \bar{h}^0(k_+, m_+), \tag{36}$$

$$\underline{h}^1(k,m) = \max\left\{\max_{\tau^k,\tau^n} u(c,n) + G(g) + \beta \underline{h}^0(k_+, m_+), \tilde{h}(k)\right\}, \tag{37}$$

$$\tilde{h}(k) = \max_{\tau^k,\tau^n}\left\{\min_{c,n,k_+,m_+} u(c,n) + G(g) + \beta \underline{h}^0(k_+, m_+)\right\}, \tag{38}$$

such that the vector $\left(\tau^k, \tau^n, c, n, k_+, g, w, r, \{m_+, h_+\}\right)$ *is admissible with respect to* $\hat{\mathbf{W}}$ *at* (k)*. We also define* $\underline{h}(k,m) = -\infty, \bar{h}(k,m) = +\infty$ *if no such vector exists.*

The above simply re-states Proposition 2. Below we briefly explain our algorithm and we refer to Feng et. al. (2011) for details in terms of approximating compact-valued set.

The numerical method proceeds as follows. Let $\mathbf{K} \times \mathbf{M} \times \mathbf{H}$ be the space of all equilibrium state vectors (k,m,h). First we define a grid $\hat{\mathbf{K}} = \{k^{i_1}\}_{i_1=1}^{N_k}$. After this discretization, instead of a correspondence $\mathbf{W}: \mathbf{K} \rightarrow \mathbf{M} \times \mathbf{H}$ we have $\hat{\mathbf{W}}: \hat{\mathbf{K}} \rightarrow \mathbf{M} \times \mathbf{H}$. It is equivalent to think about this correspondence as N_k set $\hat{\mathbf{W}}(k^{i_1})$, where $k^{i_1} \in \hat{\mathbf{K}}$. $\hat{\mathbf{W}}$ approximates $\mathbf{W}$ well as N_k goes to ∞.

We start the algorithm with an initial guess $\mathbf{W}^0(k) = \{(m, h\,(k,m))\}$ and a pre-determined tolerance $\epsilon > 0$ and follow the next steps:

- Step 1-1: given k, pick $(m,h) \in \mathbf{W}^0(k)$. We store the pair of (m,h) in $\Omega(k)$ if there exists $(\tau^k, \tau^n) \in T$, such that the pair (m_+, h_+) solves

$$h = u(c,n) + G(g) + \beta h_+, \tag{39}$$

$$u_c(c,n) - \beta m_+ = 0, \tag{40}$$

and $(m_+, h_+) \in \mathbf{W}^0(k_+)$, where (c, n, k_+) are determined as solutions to the system

$$\begin{aligned} m - u_c(c,n) \cdot \left[1 + (1-\tau^k)(r-\delta)\right] &= 0, & (41)\\ -u_n(c,n) - (1-\tau^n) w \cdot u_c(c,n) &= 0, & (42)\\ (1-\tau^n) wn + \left[1 + (1-\tau^k)(r-\delta)\right] k - (c + k_+) &= 0, & (43) \end{aligned}$$

and g, w and r are determined by (22)-(24).

- Step 1-2: given k, and $\Omega(k)$, denote $\Omega^m(k) := \{m | (m,h) \in \Omega(k)\}$, we define

$$\bar{h}^1(k,m) = \max_{\tau^k, \tau^n} \max_{(m_+,h_+) \in \mathbf{W}^0(k_+)} u(c,n) + G(g) + \beta \bar{h}^0(k_+, m_+) \tag{44}$$

$$\underline{h}^1(k,m) = \max_{\tau^k, \tau^n} \min_{(m_+,h_+) \in \mathbf{W}^0(k_+)} u(c,n) + G(g) + \beta \hat{\underline{h}}^0(k_+, m_+) \tag{45}$$

where
$$\hat{\underline{h}}^0 = \max\left\{ \underline{h}^0(k_+, m_+), \frac{1}{\beta}(\tilde{h}^0(k) - u(c^1, n^1) - G(g^1)) \right\}$$

with c^1, n^1, g^1 being determined by the solution to (45), for all $m \in \Omega^m$. Otherwise we set $\bar{h}^1(k,m) = +\infty$ and $\underline{h}^1(k,m) = -\infty$. Finally, we find

$$\tilde{h}^1(k) = \min_{m \in \Omega^m} \underline{h}^1(k,m). \tag{46}$$

- Step 2: we define $\mathbf{W}^1(k) = \left\{(m,h) | m \in \Omega^m(k), h \in \left[\underline{h}^1(k,m), \bar{h}^1(k,m)\right]\right\}$.
- Step 3: we set $\mathbf{W}^* = \mathbf{W}^1$ if $\| \mathbf{W}^1 - \mathbf{W}^0 \| < \epsilon$. If not, we set $\mathbf{W}^0 = \mathbf{W}^1$ and restart from step 1.

6.4 Building Strategies

In this Section, we outline how to find a strategy that supports a sustainable equilibrium. For a better exposition, we focus on the strategy that yields the highest payoff for the government. This procedure can be generalized to find strategies supporting any other point that belongs to the equilibrium set.

1. At $t = 0$, k_0 is given, we find the highest possible value of $h_0 = \sup\{h | (m_0, h_0) \in \mathbf{W}^*(k_0)\}$ and its corresponding m_0. Then we find the government's tax policy that can support (m_0, h_0).

More specifically, we find (τ_0^k, τ_0^n) such that

$$u(c_0, n_0) + G(g_0) + \beta h_1 = h_0 \tag{47}$$

where $h_1 = \bar{h}(k_1, m_1)$, $m_1 = \frac{u_c(c_0,n_0)}{\beta}$ and we can find the values of (c_0, n_0, k_1) by solving the following equation system

$$\begin{aligned} m_0 - u_c(c_0, n_0) \cdot \left[1 + (1 - \tau_0^k)(r_0 - \delta)\right] &= 0, & (48)\\ u_n(c_0, n_0) - (1 - \tau_0^n) w_0 \cdot u_c(c_0, n_0) &= 0, & (49)\\ (1 - \tau_0^n) w_0 n_0 + \left[1 + (1 - \tau_0^k)(r_0 - \delta)\right] k_0 - (c_0 + k_1) &= 0, & (50) \end{aligned}$$

when the values of $(\tau_0^k, \tau_0^n, m_0)$ are given. Therefore the above problem is well-defined in terms of $(\tau_0^k, \tau_0^n, m_0, h_0)$.

2. At $t = 1$, k_1, m_1, h_1 are given by the solution in step 1. Now, as in step 1, we find the government policies (τ_1^k, τ_1^n) such that

$$u(c_1, n_1) + G(g_1) + \beta h_2 = h_1. \tag{51}$$

3. We repeat step 2 for $t \to T$.

The construction above reveals that any sustainable outcome has essentially a Markovian structure in the sense that, $(k_t, \tau_t^k, \tau_t^n)$ and (m_t, h_t) only depend on history ζ^{t-1} through (m_{t-1}, h_{t-1}).

6.5 Proofs

Proof of Proposition 1: The Ramsey allocation yields $-u_{n,t}(1 + \lambda_t(1 + \chi)) = u_{c,t} f_{n,t}(1 + \lambda_t(1 - \sigma) + (\lambda_t - \lambda_{t-1})\Gamma_t - \Psi_t)$ through conditions (14) and (15). This together with the competitive equilibrium conditions (5) and (7), $\tau_t^n = 1 + \frac{u_{n,t}}{f_{n,t} u_{c,t}}$, implies the above labor tax rates. ■

Proof of Proposition 2: By definition, $\bar{h}(k, m)$ is the maximum value of h at given (k, m). Therefore,

$$\begin{aligned}\bar{h}(k,m) &= \max_{\tau^k,\tau^n}\max_{m_+,h_+} u(c,n)+G(g)+\beta h(k_+,m_+) \\ &= \max_{\tau^k,\tau^n} u(c,n)+G(g)+\max_{\tau^k,\tau^n}\max_{m_+,h_+}\beta h(k_+,m_+) \\ &= \max_{\tau^k,\tau^n} u(c,n)+G(g)+\beta\bar{h}(k_+,m_+).\end{aligned}$$

where the first equality follows from the definition of $\bar{h}(k,m)$, the second equality follows from the fact that there exists at most one pair of (c,n,k_+) consistent with $\left\{\left(\tau^k,\tau^n\right),m\right\}$ at given k. The last equality uses the definition of $\bar{h}$.

A similar argument applies to $\underline{h}(k,m)$. A few comments go as follow. First,

$$\underline{h}(k,m) = \max_{\tau^k,\tau^n}\min_{m_+,h_+} u(c,n)+G(g)+\beta h_+.$$

Secondly, it should be noted that the value of $u(c,n)+G(g)+\beta\underline{h}(k_+,m_+)$ at given $\left\{\tau^k,\tau^n\right\}$ may be lower than $\tilde{h}(k)$, which says that the incentive constraint is not satisfied when the government has the worst continuation value. When this happens, the continuation value must be increased so that the incentive constraint is satisfied. However, the corresponding payoff for the present government cannot be higher than $\tilde{h}(k)$. This is because only the *minimization* operates when $\left\{\tau^k,\tau^n\right\}$ is given. There always exists $h_+\in\left[\underline{h}(k_+,m_+),\bar{h}(k_+,m_+)\right]$ to bind the incentive constraint when the worst continuation value breaks the incentive constraint. Otherwise m should not belong to the equilibrium set. $\tilde{h}(k)$ is the payoff of the worst, it must be in the lower boundary $\underline{h}(k,m)$. As it is the worst of all, it must be equal to $\min_m \underline{h}(k,m)$. ■

6.6 Additional Tables

Table 6: Benchmark Economy: Initial Steady State

	χ	h_0	$\bar{k}$	$\bar{n}$	$\frac{k}{y}$	$\frac{\bar{c}}{y}$	$\frac{\bar{g}}{y}$	$\bar{\tau}^k$	$\bar{\tau}^n$	γ_n (calibrated)
Benchmark	0.32	-2.993	1.042	0.23	2.71	0.593	0.190	0.271	0.237	5.905
	0.50	-2.916	1.042	0.23	2.71	0.593	0.190	0.271	0.237	7.694
	1.00	-2.775	1.042	0.23	2.71	0.593	0.190	0.271	0.237	16.042
	1.50	-2.690	1.042	0.23	2.71	0.593	0.190	0.271	0.237	33.451

Table 7: Planner's Solution

	χ	h_0	$\bar{k}$	$\bar{n}$	$\frac{k}{y}$	$\frac{\bar{c}}{y}$	$\frac{\bar{g}}{y}$
Planner	0.32	-2.947	1.542	0.29	3.005	0.570	0.190
	0.50	-2.875	1.499	0.28	3.005	0.570	0.190
	1.00	-2.743	1.423	0.27	3.005	0.570	0.190
	1.50	-2.664	1.380	0.26	3.005	0.570	0.190

Table 8: Economy A

	χ	h_0	$\bar{k}$	$\bar{n}$	$\frac{k}{y}$	$\frac{\bar{c}}{y}$	$\frac{\bar{g}}{y}$	$\bar{\tau}^k$	$\bar{\tau}^n$
Ramsey	0.32	-2.986	1.200	0.226	3.009	0.566	0.190	0.00	0.288
	0.50	-2.909	1.199	0.226	3.006	0.568	0.190	0.00	0.288
	1.00	-2.769	1.199	0.227	3.002	0.570	0.190	0.00	0.288
	1.50	-2.685	1.201	0.227	3.001	0.571	0.190	0.00	0.288
Best Sustainable Equilibrium	0.32	-2.987	1.135	0.227	2.900	0.580	0.188	0.122	0.262
	0.50	-2.910	1.159	0.227	2.932	0.579	0.187	0.079	0.270
	1.00	-2.770	1.170	0.227	2.940	0.579	0.190	0.067	0.270
	1.50	-2.686	1.183	0.228	2.937	0.570	0.194	0.042	0.275
Worst sustainable Equilibrium	0.32	-3.365	0.201	0.252	0.860	0.688	0.250	0.900	0.001
	0.50	-3.247	0.198	0.250	0.830	0.688	0.243	0.895	0.001
	1.00	-3.060	0.186	0.244	0.829	0.687	0.250	0.895	0.001
	1.50	-2.930	0.183	0.242	0.829	0.685	0.250	0.895	0.001

Table 9: Economy B

	χ	h_0	$\bar{k}$	$\bar{n}$	$\frac{k}{y}$	$\frac{\bar{c}}{y}$	$\frac{\bar{g}}{y}$	$\bar{\tau}^k$	$\bar{\tau}^n$
Ramsey	0.32	−2.993	1.193	0.225	3.005	0.566	0.190	0.00	0.288
	0.50	−2.915	1.196	0.226	3.005	0.568	0.190	0.00	0.288
	1.00	−2.773	1.201	0.227	3.005	0.570	0.190	0.00	0.288
	1.50	−2.688	1.205	0.227	3.005	0.571	0.190	0.00	0.288
Best Sustainable Equilibrium	0.32	−2.998	1.194	0.225	3.006	0.571	0.188	0.00	0.285
	0.50	−2.919	1.196	0.226	3.005	0.571	0.188	0.00	0.285
	1.00	−2.777	1.201	0.226	3.005	0.571	0.189	0.00	0.286
	1.50	−2.690	1.205	0.227	3.005	0.570	0.189	0.00	0.287
Worst sustainable Equilibrium	0.32	−3.223	1.234	0.234	3.004	0.740	0.016	0.00	0.025
	0.50	−3.149	1.232	0.233	3.005	0.733	0.025	0.00	0.041
	1.00	−3.010	1.220	0.232	3.005	0.720	0.039	0.00	0.059
	1.50	−2.910	1.223	0.231	3.006	0.718	0.043	0.00	0.065