Executive Summary

Following the extreme events observed on the financial markets in the past forty years, the call for stronger regulation of financial institutions was not left unanswered as the Basel Accords were introduced and continually tightened. To determine what amount of risk capital is deemed adequate for financial institutions this framework utilises so-called risk measures. In practice, the most popular measure for downside market risk is the value at risk (VaR) and it is thus essential that it can be estimated in a way that neither extremely underestimates nor overestimates potential future losses. In literature, a variety of different models have been proposed for this task, many of which revolve around a generalized autoregressive conditional heteroskedasticity (GARCH)-type structure. These models can further be divided into univariate approaches, which only consider the portfolio returns, and multivariate procedurs, which model the joint dynamics of the constituents of the portfolio.

We revisited Fortin et al. (2022) who introduced a multivariate model class which assumes that the main risks of a portfolio can be captured in the factors of the Carhart four-factor model. Therefore, they fitted a multivariate GARCH model to the factor returns and tried to capture the multivariate non-normality and asymmetric tail dependence, which are inherent in weekly factor returns, by using a copula that links the so obtained filtered standardized residuals. After some more intermediate steps, one obtains a simulated vector of one-period-ahead portfolio returns from which it is straightforward to calculate the VaR. This approach, albeit very interesting, did not manage to yield adequate VaR forecasts for a portfolio of stocks and moreover did not outperform simpler univariate models in terms of predictive ability.

Consequently, we analysed a long portfolio consisting of the same ten large cap stocks that Fortin et al. (2022) used in their paper but made some adjustments to their setup. Most notably, we considering daily instead of weekly factor and stock returns based on the findings of Kole et al. (2017) that daily returns lead to better VaR forecast than returns with a lower frequency. When inves-

tigating these daily return series we found compelling evidence for the univariate and multivariate non-normality of both stock returns and factor returns. Moreover, we found clear signs of volatility clustering in the portfolio, the single stocks and the factors justifying the usage of GARCH models.

In our empirical application, we examined whether there is a distinct advantage in using a complex multivariate model over a more parsimonious univariate approach, or vice versa. Hence, we replicated the model class described in Fortin et al. (2022) and compared it with models which are more established in literature. For our univariate models we considered the EWMA as recommended in J.P. Morgan (1996), the GJR-GARCH suggested in Glosten et al. (1993), the NGARCH as introduced in Engle and Ng (1993) and the diagonal MixN(k)-GARCH by Haas et al. (2004). Additionally, we included a second multivariate model class in form of the COMFORT model by Paolella and Polak (2015). Finally, we used the normal GARCH as a univariate benchmark and the multivariate normal DCC-GARCH as a multivariate benchmark. The resulting VaR forecasts were then backtested for desirable properties using the violation-based likelihood ratio tests of Christoffersen (1998) and compared in terms of forecasting ability by means of the conditional predictive ability test of Giacomini and White (2006).

We find that the VaR forecasts of almost all univariate models we considered are inadequate, while the multivariate models have few problems passing these backtests. However, we do not find evidence that the multivariate models systematically outperform their univariate counterparts with regards to predictive accuracy, or vice versa. We extend the existing literature by showing that the multivariate models of Fortin et al. (2022) produce appropriate VaR forecasts for a portfolio of stocks when daily instead of weekly data is used. We further introduced our own modified version of the factor copula model of Fortin et al. (2022) that uses a normal GARCH model for the marginals instead of the skewed-t NGARCH models these aforementioned authors used. Moreover, we discover that these adapted models generate better VaR forecasts than the unmodified versions.

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