Heterogeneous Tail Multivariate Financial Asset Returns Modeling

Master's Thesis — Executive Summary

Submitted in partial fulfillment of the requirements for the degree of Master of Arts in Economics and Business Administration

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Date of Submission: August 24, 2022

Executive Summary

A subset of the (Heterogeneous-) Multivariate Normal Mean-Variance Mixtures, the *Marginally Endowed Student's t Independent*, coupled with an APARCH(1,1) structure, is being introduced with the aim of providing a method for portfolio optimization. The K-variate probability density function of the used distribution with its K-fold integral representation nullifies the possibility of an analytical solution; however, sampling from this multivariate structure is fast. This will allow an empirical representation of the distribution, which can then be used for determining the optimal asset allocation. The optimization criteria for portfolios will include the mean and Expected Shortfall (ES) of the distribution, which, with the sampling approach, can now be evaluated empirically. With that however, as in every sampling based approach, the question arises of how much samples are required for the resulting empirical distribution to be stable. In the limiting case as the number of samples goes to infinity, the empirical distribution converges to the true distribution. This, however, due to computational limitations, is neither feasible nor necessary. The goal is hence to find the optimal number of samples, keeping in mind the trade-off between accuracy and computational efficiency, where this number depends positively on the average predictive scale term from the APARCH structure. Having determined a reasonable empirical distribution, the next step is to define the optimization criteria for the portfolio. Given the empirical mean and ES, two approaches will be considered. First, the *Max Mean-ES* optimization, as the name suggests, maximizes the risk-return ratio of mean to ES. Second, the *Mean-ES* optimization minimizes the risk, i.e. the ES, for a given predefined expected return — similar to the Markowitz approach.

With the empirical distribution of the returns, also the optimization itself will be be conducted in a similar way, where we restrict ourselves to a fully invested long only portfolio. With the ability to draw such a weight vector randomly from a K-dimensional simplex, a set of weight vector samples can be drawn, from which the best one with respect to the defined optimization criteria is taken as the optimal portfolio vector. We can think of the weight vector as a composition of n discrete weights of size — i.e. of granularity — $1/n$ that can be assigned to the portfolio consisting of K stocks. This leaves us with a combinatorial exercise similar to an integer composition, which reveals

that the simplex under consideration needs to be limited in order for the sampling based approach to be valid. Otherwise, only a fraction of the possible weight combinations would have been considered in the first place and hence the empirically optimal solution would be invalid. The necessity to limit the simplex yields three approaches, which give rise to several portfolio strategies. First, one can only sample very close around the centre of the simplex, leading to a local optima around the equal weight portfolio. Second, upon reducing the granularity of the weights (small n), one samples at the corner of the simplex and most of the weight is assigned to just a small number of stocks. Lastly, provided that the portfolio has a limited number of stocks, it is possible to sample uniformly over the limited simplex which results in a global solution. For each of these strategies the number of weight samples required for a valid solution, i.e. one leading to a stable return sequence that is repeatable, was determined.

A huge drawback of the empirical approach for portfolio optimization is its high demand for computational resources. The codes were implemented in Matlab, where the proposed method lends itself to fast vectorized code involving huge matrix multiplications in order to obtain the desired predictive *portfolio* returns. This however requires an immense amount of Random Access Memory (RAM), which can be overcome by partitioning the matrices into several smaller matrices that can each be handled with the available RAM. Consequently, this partly reverses the vectorization and its efficiency advantage, but it will be necessary and is still faster than non-vectorized code. Next, to tackle the runtime of the matrix operations, codes are adapted such that specific operations can be outsourced to the Graphics Processing Unit (GPU), which is specialized in matrix operations, leading to an enormous efficiency gain. Next, with the optimization being based on random and, crucially, independent samples, calculations can be addressed in parallel. While parallel processing is already provided upon using vectorized code, this built-in parallelization does not work with the complementary GPU. Hence, a last step in computational improvements was the adaption to parallel processing of the method with several GPUs, leading to a computational efficiency gain nearly one to one with the number of available GPUs.

Having implemented a fast way of running a sample based optimization, one could finally turn to a large-scaled backtesting of the proposed strategies. For each, a portfolio consisting of the current Dow Jones Industrial Average stocks is being constructed and is tested against several benchmark portfolios such as Markowitz and Max-Sharpe, next to the simple equal weight portfolio. For the benchmark portfolios, the same stocks, i.e. the current index stocks, have been used in order to allow for a comparison. However, a comparison to the actual index performance during the same time period will be misleading because of the survivorship bias. The period of 2016-2020 is being studied, with a total of 1'100 trading days, where high volatility periods have to be studied separately due to the requirements of the sample based optimization. The backtesting analysis showed that the Max Mean-ES usually performs better than the Mean-ES optimization with an expected annual return of 10%. The analysis revealed that two out of three approaches have a promising return structure, namely sampling at the corner of the simplex to assign weight to just a small number of stocks, and the uniform sampling over a limited number of stocks. For the latter, a method for determining this subset of stocks has yet to be developed and is hence subject to future research. Lastly, the approach of sampling close to the equal weight portfolio does not lead to an outperformance and just follows the regular equal weight portfolio very closely, though with a higher turnover which further reduces performance, as becomes apparent in the net return analysis. The same issue, high turnover, also applies to the two other approaches, in fact even more. It is however important to highlight that daily rebalancing was used and the smallest improvement with regard to the optimization criteria triggered a rebalancing. Hence, upon introducing restrictions to the rebalancing, the high turnover can be addressed and outperformance in net returns might be achieved. Those are, for example, a lower rebalancing frequency in general and/or minimal improvements in the optimization criteria required to trigger a rebalancing. This is also subject to future research and was not further addressed in the framework of this thesis. With the magnitude of the outperformance in gross returns compared to any of the benchmark portfolios, there is a substantial amount of turnover that can be accepted and still leads to an outperformance, provided the gross performance remains reasonably stable upon introducing the proposed rebalancing restrictions.