Executive Summary

In this master’s thesis we study the pricing and hedging of multi-asset equity options with emphasis on modelling the correlation between the underlyings. We are mostly interested in one specific such option, that is the multi-barrier reverse convertible (multi for short). It is worldwide a flagship structured product. Multi’s are not only over multiple stocks, they are also path-dependent and have a worst-of feature. These make them very sensitive to correlation changes and their pricing is tricky. We would like to answer four questions in this master’s thesis.

Q1: Which correlation model is practical and appropriate for multi’s?

A1: In Chap. 2 we review some main correlation models and highlight their strengths and weaknesses. We choose eventually the Jacobi process. It is a mean-reverting bounded uni-variate stochastic process. It has the following advantages. Firstly, its stochasticity offers great flexibility for trader’s judgment on the trend of correlation over the simulation horizon; thereby, it does not suffer the potential underpricing problem of deterministic correlation models, especially in financial crises. Secondly, as it’s a uni-variate process, its calibration stays relatively simple (see Chap. 3.3). Thirdly, we developed a fast pricing technique using Graphics Processing Unit (GPU) which makes the implementation of the model realistic (see Chap. 3.1).

Using the Jacobi process and the fast GPU pricing technique it is straightforward to define and compute multi’s correlation exposure. We answer Question 2 in Chap. 3:

Q2: How to measure multi’s correlation exposure?

A2: We define it as the multi’s price change with respect to the initial value of the Jacobi process\(^1\).

\(^1\) In contrast, a definition is trickier if a correlation matrix with \(\frac{n(n-1)}{2}\) a priori different pairwise correlations is used.
The definition and computation of multi’s correlation exposure could be applied in the same way to other multi-asset equity options. Hence correlation hedging becomes conceptually trivial. That is by matching their correlation exposure. But before we implement it we discuss potential hedging instruments.

Q3: What are potential instruments and trading strategies for hedging correlation and which one should we choose for multi’s?

A3: In Chap. 4 we describe correlation swaps and dispersion trades. Among them, a Call-versus-call (CvC) strategy on baskets is the simplest and the most liquid correlation hedging strategy. However, it does not provide pure correlation exposure, therefore the hedging of other sensitivities of the portfolio needs to be adjusted accordingly.

Finally we demonstrate how correlation can be hedged under the Jacobi correlation model. We use CvC to match multi’s correlation exposure and delta hedge both the multi and the CvC. This answers the last question:

Q4: How to use correlation hedging strategies and does it effectively reduce risk?

A4: Assuming zero vega (sensitivity to volatility change) and zero theta (sentitivity to time change), compared with pure delta hedging, CvC-delta hedging (i.e. correlation combined with delta hedging) can reduce significantly the hedging P&L fluctuations (see Chap. 5).

This master’s thesis brings three novelties to the literature:

1. For the first time the Jacobi correlation model is applied to multi-barrier reverse convertibles. We also propose an original way for calibrating the starting value of the Jacobi process.

2. We developed a GPU simulation technique which tremendously speeds up pricing and hedging under stochastic correlation model.

3. We give an empirical example of how correlation can be hedged under the Jacobi correlation model.

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In other words, being one-dimensional, the Jacobi correlation model makes the definition of correlation exposure very simple.
All the above is carried out within well defined scope. Firstly, we do not model FX and assume that all underlyings are in one and the same currency. Secondly, we consider multi’s as single instruments, and not books of multi’s. Thirdly, we use Black-Scholes model for each underlying (as marginals). That also means we do not model stochastic volatility. Instead, volatility is supposed to be the implied volatility (hence deterministic over the simulation horizon). This is more for the simplicity of the master’s thesis rather than due to real technical limitations. In fact, the Jacobi correlation model can be used alongside other volatility models.