Executive Summary

Problem statement

Over the past decade the notion of volatility risk and the demand for its hedging materialized in rapid increase in trading activity in variance derivatives. One of the popular instruments for mitigation of the variance risk are VIX (S&P 500 implied volatility index) options. Emergence of a new asset class necessarily calls for stochastic models capable of precise, efficient pricing and replicating the properties of dynamics of the underlying assets. The task at hand is to identify the main characteristics of the driving process and to formulate a model incorporating all the features. To achieve this goal, the evidence from the literature is complemented by empirical analysis addressing questions of the presence of jumps, stochastic volatility of volatility, optimal number of factors and time-dependence property.

Empirical evidence

The evidence of long memory in VIX found in this thesis is in line with common conclusion in the literature. Even further, the tests of long memory imply that not only the VIX is characterized by long-range dependence, but also its volatility. Next, a non-parametric analysis of jumps using high-frequency data identifies frequent jumps, predominantly occurring simultaneously in VIX and its volatility. A panel of option data further suggests that, in contrast with the literature, affine jump-diffusion framework is not an accurate description of the dynamic nature. On the contrary, the results are compatible with the presence of long-memory factors.

Model formulation

Most of the documented properties of the VIX dynamics could be readily modelled in an affine framework. Unfortunately, standard affine jump-diffusion cannot achieve long-range dependence. So called supOU processes represent a class of processes which allows for long memory. This thesis presents a new model building upon supOU processes - three-factor long-memory (3FLM) model. The model is designed to have both idiosyncratic and common jumps for VIX with its volatility,

long-range dependence in both and three factors driving the VIX.

Pricing and empirical performance

The 3FLM model shares one of the virtues of standard affine models - its characteristic function can be derived in closed form, up to computation of integrals appearing in it. As a consequence, pricing methods based on the knowledge of characteristic function are straightforward to adapt. To this end, COS-method together with approximation schemes for the integrals is a suitable approach to pricing.

Calibration of the model to the option data demonstrates reasonable performance for the pricing. At the same time, it highlights the important areas of future research - development of more efficient calibration procedure and solving the estimation task, which would allow to fully assess the merit of modelling long memory in VIX and its volatility.