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Pricing of Energy Commodity Derivatives

Master thesis
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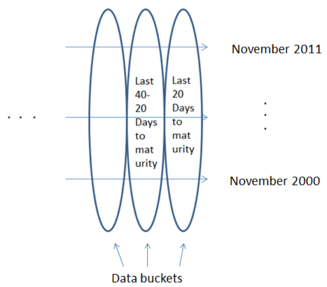
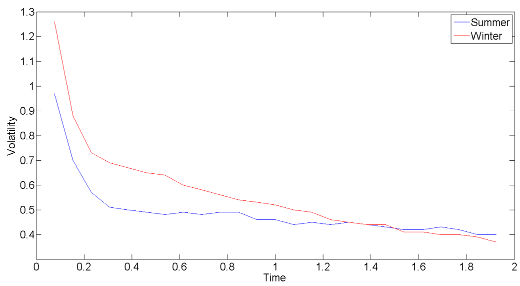
Executive Summary

This master thesis about *Pricing of Energy Commodity Derivatives* focuses on analysing the seasonal stochastic behaviour of natural gas, heating oil and gasoline. First we present and explain the nature of the American futures option contract and present the underlying and the contractual obligations arising from futures contracts, like the physical delivery rules.
Then we present previous results and models specified for pricing of Energy commodity derivatives or for pricing other underlyings with seasonal behaviour. Later we will modify the model proposed by Back et al. [4] and introduce a slight modification in the handling of seasonality.
Futures price data of the three underlyings are analysed by maturity and time-to-maturity clustering. The logarithmic returns are grouped by maturity and by 20 business days clusters. For example the first November expiry basket contains the logarithmic returns of the last twenty business days of trading for all the 12 November expiries in our dataset. The figure below should clarify this. Based on the returns in these baskets the volatility is calculated. We then analyse these volatility baskets with respect to time-to-maturity and can build two groups with distinct volatility functions, like a group for summer or winter maturities.



(a) Data buckets



(b) Natural gas futures price volatility functions

These functions are usually convex and rise with reducing time-to-maturity. Typically one can distinguish the volatility scheme with less than one year time-to-maturity, the effect gets pronounced with less than six months time-to-maturity. We take the average for each group as the volatility function for the whole group and will use that later in our proposed model.
Finite elements as a technique to price European calls and puts is introduced with the Black & Scholes model as example. Then the difference between the American and European style exercise is accounted for by introducing the partial differential inequation which we transform into a linear complementarity problem and then numerically solve by using the quasi-newton approach.
We present our model to tackle the findings of the previous chapters. The model is basically the same as proposed by Back et al. [4] but we will introduce and use a different functional form of the seasonal volatility function. Let us introduce the values first. The futures price is equal to F_t and the volatility or the variance equal to V_t depending on the value of c_2. The values of $c_1 = 1$ and $c_3 = \frac{1}{2}$ represent typical choices for such a model with mean reversion. W is a two dimensional standard Brownian motion with zero correlation. κ is the speed of mean reversion, θ the seasonal volatility function and ρ the correlation between the volatility or variance process and the futures price.

$$
\begin{aligned}
dF_t &= {F_t}^{c_1} {V_t}^{c_2} d{W_t}^1 \\
dV_t &= (\kappa(\theta_t - V_t) - \lambda V_t)\, dt + \sigma {V_t}^{c_3} (\rho d{W_t}^1 + \sqrt{1-\rho^2} d{W_t}^2)
\end{aligned}
\tag{0.1}
$$

This model should capture the observed ascent of futures price volatility with decreasing maturity if we use the previously fit seasonal volatility functions. Furthermore the speed of mean reversion can influence the stickiness to the seasonal mean and therefore differentiate the prices under the same set of parameters for the whole term-structure as the seasonal mean parameter is not constant. For example

when we start with a low starting volatility parameter and have an increasing seasonal volatility function which crosses the starting volatility level at half of the time interval analysed, then an increase in speed of mean reversion will increase the short term values and decreases the long dated option prices. This will prove to be a handy feature of the model. The correlation is another parameter which governs the shape of the volatility surface, and helps to fit the model to market prices.
The knowledge of Greeks for such a model is essential for different purposes, they prove to be handy for calibration, risk management and hedging. So we will derive the Greeks with respect to model parameters and solution arguments. We comment the shape of the Greeks with respect to economical arguments and compare them across different pay-offs and exercise styles. Later we will use these Greeks to calibrate the model.
To calibrate the model we will use a least squares algorithm. We will minimize the norm of the difference between the observed market prices, the model prices and the change in parameters times the respective Greek. We will use call and put option market data of options being between zero and ten percent in the money. We allow the parameters to fluctuate between certain boundaries. With this procedure we fit the speed of mean reversion, the volatility of volatility, the correlation and for each maturity analysed, a separate starting volatility value. This approach will converge very fast to an accurate solution as the Greeks contain high quality information about the price changes induced by a parameter change. Usually we reach a relative root mean squared error of around 0.5-2.5% after one iteration taking the previous days parameters set as an initial guess, for heating oil and gasoline. The root mean squared error (RMSE), with transformed option prices with strike equal to one, is around 10e-4. For gasoline we split the dataset in calls and puts due to significant premium differences. For calls we have RMSE of around 5e-4 and for puts we have RMSE of around 13e-4. We then discuss the parameters and their behaviour over a time set of 69 trading days where we fitted the term structure. One drawback of the calibration is certainly the time used to run the finite elements algorithm, since one has to evaluate put and calls with different seasonality function, which will lead to computing times of around eight minutes on a modern personal computer for one iteration. The least squares algorithm then takes just a couple of seconds, so there is a gap between computational expenses to provide the data for the minimization problem, and the minimization itself.
We conclude that the model proves to be useful to price American futures options on energy commodities accurately. The knowledge of the Greeks and the reliable calibration to market data makes the model directly applicable to real world applications.