Value-at-Risk, Tail Value-at-Risk: a comparative study

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Abstract

The goal of this paper is to make a comparison between the risk measures Value-at-Risk (VaR) and Tail Value-at-Risk (TVaR). We start by formulating a suitable framework and making assumptions on the possible financial positions. Besides the mathematical formulation of the results, there will be a short financial interpretation.

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1 Notation

1.1 Framework

Definition 1.1 (Probability space)**.** We consider a *probability space* $(\Omega, \mathcal{F}, \mathbb{P})$, where

- Ω is a non-empty, possibly infinte, set which models all future states of the world;
- $\mathcal{F}$ is a *σ-algebra* i.e. a family of subsets of Ω such that:
  - $\Omega \in \mathcal{F}$,
  - $A \in \mathcal{F} \Rightarrow A^c \in \mathcal{F}$,
  - $(A_n)_{n\in\mathbb{N}} \subset \mathcal{F} \Rightarrow \bigcup_{n\in\mathbb{N}} A_n \in \mathcal{F}$;
- $\mathbb{P}$ is a *probability measure* on $(\Omega, \mathcal{F})$ i.e. a mapping $\mathbb{P} : \mathcal{F} \to [0, 1]$ such that $\mathbb{P}(\Omega) = 1$ and for every sequence $(A_n) \subset \mathcal{F}$ of pairwise disjoint sets: $\mathbb{P}(\bigcup_{n=1}^{\infty} A_n) = \sum_{n=1}^{\infty} \mathbb{P}(A_n)$.

In the rest of this paper, we fix $(\Omega, \mathcal{F}, \mathbb{P})$ as our probability space.