Expectations Implied in Option Prices: An Equilibrium Approach

by

Sandro Caluori
Supervisor: Prof. Dr. Thorsten Hens

A thesis submitted to the Faculty of Economics
in partial fulfillment of the requirements for the
degree of Master of Arts in Banking and Finance

University of Zurich,
Zurich, Switzerland
December 2011

Matriculation number: 05-907-324

Executive summary

The pricing kernel is an essential quantity in financial economics. In classical equilibrium models, it links the prices on the market with the preferences of the agents. Assuming that there is a risk-averse representative agent, the pricing kernel is closely related to the marginal utility of the representative agent via the following relationship:

$$\text{marginal utility} \propto \frac{\text{risk-neutral probability}}{\text{subjective probability}}.$$

Given a complete market and no arbitrage opportunities, the risk-neutral probability is the probability measure under which the expected value of future outcomes of an asset equals its price. Once we have obtained the risk-neutral probability, every asset can be priced. The subjective probability expresses the opinion of an investor of how likely a state is to occur. Considering the ratio of the risk-neutral and the subjective probability, called the pricing kernel, we obtain in a complete, arbitrage-free market with a single representative agent the above-mentioned relationship to the marginal utility of the representative agent. We gain thereby information about his risk behavior and his preferences. The pricing kernel is in the proper sense nothing but a risk aversion adjustment. In order to extract the risk aversion adjustment from the market, there are plenty of techniques to estimate the pricing kernel. One often used approach is to estimate the two probabilities separately in order to divide the risk-neutral by the subjective probability afterwards. Breeden and Litzenberger developed in 1978 a method to estimate the risk-neutral probability out of option prices. Considering options on a large index like the S&P 500 or the DAX fulfills the assumption of complete markets because every desirable payoff may be constructed by changing the strike price alone. Further it is possible to investigate a whole economy, since a large index covers a large part of the economy. This method is considered as a robust technique which is rather less subject to estimation risk. The subjective probability, however, is often approximated by historical return data, which leads to more estimation risk.

Since it is questionable to reason the future from the past, this paper investigates a new approach. During stable times the historical approach may work. But especially in volatile periods, at the beginning of a crisis or at the beginning of recovery the historical prediction accuracy is poor. What this paper

investigates is an estimation of the subjective probability without consideration of the history. Instead it considers the current environment and assumes preferences of the market. In particular I make use of the relationship above and conclude the subjective probability by dividing the risk-neutral probability by the marginal utility. All I need is an estimation of the risk-neutral probability, obtained by the method of Breeden and Litzenberger, and an assumption of the form of the utility function. In various test I then investigate which probability, the historical or the utility assumption one, better fits the real probability. Since the step of assuming the marginal utility is a very crucial one I consider different set ups and take account of the pricing kernel puzzle which excludes the assumption of fully risk-averse agents.

The results are not ambiguous. The investigation period is from the beginning of 2001 to the beginning of 2002 on option price data on the DAX and its return data. Considering a single, risk-averse representative investor, the historical probability distribution usually better describes the real distribution than the utility assumption approach does. However, the performance of the utility assumption approach depends on the level of risk aversion and the time to maturity. The lower the level of risk aversion is, the better the approach works. The shorter time to maturity, on the other hand, the worse the approach performs. This is mainly due to the fact that even small changes have a great influence in the short term, whereas they don't have a great influence in the long run. Therefore the predictive efficiency is, against the intuition, poorer for short term. Considering single days, like low points, the utility approach works sometimes better but throughout the whole year the historical approach is closer to the real distribution. Since the pricing kernel puzzle indicate non-strictly risk-averse investors I correct, in a next step, the pricing kernel for the increasing part. Immediately the results improve and are at least as good as the historical distribution. Since the transformation of the pricing kernel is very basic, the results are remarkable. The improvement of the results confirms the existence of the pricing kernel puzzle for this data. Therefore further investigation with data on a longer period might be interesting. Also remarkable is the good performance of the historical approach. But again an investigation of the period from 2006 to 2009 might be very interesting due to the extreme market conditions in 2008. The approach of utility assumption might be useful as an additional backup method, where history does not matter and only current market conditions count.