A Real Options Approach to Managers' Investment Decisions in Games with Incomplete Information

Bachelorarbeit

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Asset Pricing and Risk Management

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bei

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Executive Summary

I Statement of the problem

This thesis work is related to real option theory, especially to its merge with game theory.
In the most relevant papers dedicated to this feld, it is considered that a firm must maximize its value, but in reality, it is its executive who makes decisions. Managers have often interests that deviates from those of shareholders.
The main idea of this thesis is that there is competition amongst managers who will be the first. We apply game theory on the interaction between managers, not firms. A manager, who made the best performance in the branch can count on the career advancement and other benefits. In order to achieve such a result one can take speculative positions, for instance, to invest earlier in a project when the situation on the market is not clear. This action increases the probability of becoming the first (if N>2), but also the probability to fail at all. In other words, he plays at the expense of the expectation of the firm's value.
In the theoretical basis of the work, a short overview was given on the topics from corporate finance, game theory and option pricing which were needed to investigate the question. We took a look there at what drives managers' motivation, why it can be profitable to wait with the investment, how preemption competition can be described with the game theory and what a business project has in common with financial options.

II Goal setting

The goal of the thesis work is to put manager's motivation and incentives, especially a wish to be the first in a branch, into the framework and investigate what impact it will have on investment decisions. The other purpose was to develop this concept theoretically.

III Approaches and methods

In the model of the market that we use the price is given by $\mathrm{P}_t = Y_t D(Q)$ in which $\mathrm{dY}_t = \mu Y_t \mathrm{d}t + \sigma Y_t \mathrm{d}\omega$
$Y_0 = y$ where $y > 0, 0 < \mu < r, \sigma > 0 \quad \mathrm{d}\omega \sim$ iind(0,dt)
D(Q) is a decreasing function representing the non-stochastic part of inverse demand function. "iind" means "identically independend normally distributed"
Because a firm can produce in our model only one unit of production, its profit stream is $1 \times P_t = Y_t D(Q)$.
Each firm decides at every point of time whether to invest or not (if it has not already invested). If a firm were the only one on the market, it would wait (this situation is mentioned in the theoretical overview of the work), but in reality it fears to be preempted by its competitors. Thus, as the value of the first investor exceeds that of the second, it would be invested.
It arises another problem: who will be that one that invests? It is the worst case that could happen, if both will invest simultaneously. It is the coordination problem. This problem was solved in the mixed strategies in the literature for the case N=2 firms. In this work I also solved it for the general case of N firms.
For the practical purpose I made a Monte Carlo simulation of the model for the case N=3 firms. It was assumed that instead of investing when expectations of values are equal, a "risky" manager can try to invest when Probability$[V_1 > V_2 = V_3] >$ q (until the first has invested the values of the second and third doesn't exist). For instance, q=0.45.
It was also supposed that only one of three managers behaves "risky" and that this one tries to come to result of the end of reporting period of 3 years. I run 100 simulations of 150 time periods (each period is 3 months). It was also assumed that the other two managers act believing that all three are "risk-neutral". Furthermore, it is supposed that this manager tries to come to result at the end of reporting period of three years.

An important step is the discretization and implementation of formulae. The

discrete Y-process takes a form of: $\mathrm{Y}_t = Y_{t-1}(\mu\Delta t + \sigma\Delta\omega + 1)$
It is a task of dynamic programming to find the investing triggers for each investor, that is, at which values of Y each of them invests. Such a kind of task is usually solved backwards.
First, one finds the optimal decision of the third investor, then, for the second (the second set the optimal solution of the third into his reflections). And at the end the first decides knowing how the third and the second will act.
I substitued the stochastic values with their expectations and then discretized the formulae.
The third investor invests as the Y-process touches the threshold: $\frac{\beta_1}{\beta_1-1}\frac{(r-\mu)I}{D(3)}$, where
$\beta_1 = \frac{0.5\sigma^2-\mu+\sqrt{(0.5\sigma^2-\mu)^2+2\sigma^2 r}}{\sigma^2} > 1$
The next step is to find the investing trigger Y_2 for the second investor, it is the solution of the following equation:

$$\frac{YD(2)}{r-\mu}\Big(1-\Big(\frac{Y}{Y_3}\Big)^{\beta_1-1}+\Big(\frac{Y}{Y_3}\Big)^{\beta_1}\frac{Y_3D(3)}{r-\mu}\Big)-I=\Big(\frac{Y}{Y_3}\Big)^{\beta_1}\Big(\frac{Y_3D(3)}{r-\mu}-I\Big) \quad (1)$$

And finally in the same way the investing trigger Y_1 for the first investor was found, it is the solution of the following equation:

$$\frac{YD(1)}{r-\mu}\Big(1-\Big(\frac{Y}{Y_2}\Big)^{\beta_1-1}\Big)+\frac{YD(2)}{r-\mu}\Big(1-\Big(\frac{Y}{Y_3}\Big)^{\beta_1-1}\Big)-\frac{YD(2)}{r-\mu}\Big(1-\Big(\frac{Y}{Y_2}\Big)^{\beta_1-1}\Big)+$$

$$+\Big(\frac{Y}{Y_3}\Big)^{\beta_1}\frac{Y_3D(3)}{r-\mu}-I=\Big(\frac{Y}{Y_2}\Big)^{\beta_1}V_2$$

Thus, in this way we find the thresholds at which the market participants invest.
Finally, we compute the threshold Y_r at which an "ambigious" manager is ready to imvest.

IV Results

I came to the result that, if the "ambigious" manager invests at the percentile 0.45, with respect to the given parameters, he would invest earlier. The computations shows that in 35 cases out of 100 (a frequency of 0.35) he becomes a winner in the firm's-value race. Thus, the "ambigious" manager reaches his goal with tricky behaviour. On the other hand, Y_r threshold is not optimal for the firm - the firm's optimum is Y_1! Thus, because of his ambitions and preferences, the manager is expected to sink the firm's value, but is likely (more likely as the other two) to win the race.