Market risk modeling and Value-at-Risk reduction

Bachelor Thesis in Quantitative Finance

Swiss Banking Institute

University of Zurich

30 July 2007

Prof. Dr. Paolo Vanini

Peter Andrés Münzhuber

Executive Summary

A portfolio optimization process can be roughly divided in two steps; the asset allocation subject to the investor's risk tolerance and investment horizon and an optimal risk measurement and management.
It is the aim of this work to explore the latter and to particularly focus on the market risk exposure each portfolio holder faces. The Value-at-Risk concept (VaR) will be used as a risk measurement and management tool.

Problem

Portfolios held by financial institutions as well as by private investors are exposed to *many sources of risk*. One specific risk is *market risk* which in terms of modern portfolio theory is also referred to as undiversifiable risk. As a result, portfolio optimization includes risk measurement as well as risk management.
A risk measurement tool imposed by the Basle Committee on Banking Supervision and therefore often applied by financial institutions is the Value-at-Risk (VaR). There are several ways to compute this number; a standard technique is the variance- covariance approach which makes the unfortunate assumption that financial returns are *normally distributed*. Empirically though, it is a well known fact that log returns do not seem to be properly fitted by the Gaussian distribution. Specifically, in the lower tail region of the density probability distribution the normal approximation is likely to underestimate the actual VaR. Further emphasis is put to the *high dimensionality* problem with regards to the covariance matrix computation for large and complex portfolios.
A third failure addressed is the *non-subadditivity* of this risk measure. Apparantly, the VaR does not seem to obey a time-honoured principle in financial economics, namely, the diversification effect.
The last problem is motivated by two different issues; firstly, it is an empirical proof of the *translation invariance* axiom in the context of coherent risk measures. The second view arises from the *minimum capital requirement* based on the "three zone approach" imposed by the regulatory body of the respective country. Therefore, there is an incentive for a financial institution to reduce its VaR number. The reduction shall be based on a hedging strategy using a European Put Option on the overall portfolio.

Procedure

Part I
The first part gives an overview of the different risk forms. In addition, it briefly addresses several risk measurement concepts often used in the financial industry.

Part II
This part specifies the role of market risk in the context of risk management. The overall market risk is decomposed in specific market variables from which the market invariants will be computed based on statistical evaluations. The

invariance property enables the inference of the probability distribution. The normal and t-distribution will be approximized to the empirical data.

Part III
The third part induces the findings of the previous part into the formal definition of the VaR for normal and t-distributed financial returns. Futhermore, an evaluation of the VaR concept is provided with regards to high dimensionality, fat tails, coherency and volatility clustering. Two weaknesses are addressed more carefully. A dimensionality reduction technique is presented based on Principal Component Analysis. Furthermore, a numerical method is applied to measure the degree heavy-tailedness of P&L distributions which gives an indication of whether the subadditivity property is satisfied or not.

Part IV
The fourth part applies the optimal hedging strategy to the Test Environment in order to reduce the VaR. Furthermore, the underlying parameters such as the strike price, percentile and the moneyness of the Put Option are explored more carefully.

Results

The results are based on a non-trivial *equity portfolio* containing 25 Swiss stocks and a *foreign exchange portfolio* of six different currencies. The market invariant for the equity and foreign exchange market is the continously compounded return. It satisfies the invariance properties with regards to be independently and identically distributed.
The comparison of the VaR estimates based on a normal and t-distribtion fit gives evidence for the t-distribution to be producing fatter tails below a left tail probability of 3% and above 97%. Consequently, the VaR estimates become more conservative in that specific lower tail region where as for the normal model it is the case for the respective intermediate values.
A solution to the high dimensionality problem is offered by Principal Component Analysis which enables the decomposition of the covariance matrix in a orthogonal and diagonal matrix. For this purpose, the covariance matrix needs to be symmetric and positive definite. Specifically, a pre-specified percentage (e.g. 90%) of both portfolios' total variances can be explained by a significantly lower number of variables; that is, for both portfolios the dimensionality could be reduced by approximately 50%.
It is often read in financial literature that the VaR does not satisfy the subadditivity property. That trend may be *tapering off* since non-subadditivity only occurs under certain circumstances. These cases are based on extremely skewed and heavy tailed P&L distributions. In the context of the equity and foreign exchange portfolio it is shown that the VaR, in fact, takes the diversification effect into account. The results are based on a tail index estimate provided by the Hill method. For both portfolios the tail index estimate turns out to lie in the region where the subadditivity is satisfied and the first moment is finite. These estimates are consistent with the ones found by other authors for other

stock returns.
Regarding the incentive to reduce the VaR, it is shown that using a single long position of a European Put Option on the overall portfolio, the VaR is reduced. The most important results found in the context of the Test Environment are consistent with the ones found by other authors:

- The amount of VaR reduced is positively and linearly dependent of the cost expenditure constraint imposed.
- The optimal strike price is independent of the cost expenditure constraint level.
- In the context of VaR, for any level of left tail probability below 10%, the optimal hedging strategy implies the purchase of the Put Option always to be out-of-the-money.
- There is empirical evidence of a concave- shaped strike price/ percentile relation in the first 10% left tail probability.

General Evaluation

It should be noted that the data set collected is primarily limited to the equity and foreign exchange market. Specifically, they represent non- trivial portfolios. Furthermore, the critical evaluation with regards to the assumed normally distributed financial returns are only valid in the context of such VaR computation techniques which actually exploit this assumption such as the variance- covariance approach. For instance, the historical simulation method does not make any distributional assumptions. Nevertheless, each approach yields at the end a VaR number for a particular left tail probability.
An extension to this work could be a further evaluation of the Expected Shortfall (ES) relative to the VaR. Particularly, to question whether it is plausible solely for reasons of non-subadditivity to adopt the ES as an alternative to the VaR. Coherent risk meausures provide an axiomatization of risks upon which the VaR has been criticized. However, it is strongly theoretically focused. This trend might be dampened since it is also known that the VaR based on the most commonly observed heavy tailed assets does actually satisfy the principle of diversification. In fact, empirical data falling into the category of non-subadditivity is hard to find. Also, it has to be considered that such data would be characterized by exreme losses in the lower tail region. Such extreme events do not happen frequently enough in order to be modelled with probability distributions.

Audience and style

This work is supported by computational software; MatLab is used for mathematical operations and simulations. Additionally, for statistical evaluations R and S-PLUS have been taken as the appropriate program. This will help the reader visualize and understand the ongoing steps throughout the paper. The detailed time series of the Test Environment and various computations are found in the CD attached.
The audience or reader, respectivley, are either practitionars or people from an academic institution. Nevertheless, the reader must be familiar with standard statistics and linear algebra.

Keywords:

Market invariant, Value-at-Risk, Subadditivity, Principal Component Analysis