The Pricing of Systematic and Idiosyncratic Variance Risk

Norman Schürhoff Alexandre Ziegler *

July 8, 2010

*Schürhoff is with the University of Lausanne, Swiss Finance Institute, and CEPR (Phone: +41 (21) 692-3447. E-mail: norman.schuerhoff@unil.ch. Postal: Ecole des HEC, Université de Lausanne, Extranef 239, CH-1015 Lausanne, Switzerland). Ziegler is with the University of Zurich and Swiss Finance Institute (Phone: +41 (44) 634-2732. E-mail: ziegler@isb.uzh.ch. Postal: Institut für Schweizerisches Bankwesen, Universität Zürich, Plattenstrasse 14, CH-8032 Zürich, Switzerland). We thank Bernard Dumas and Amit Goyal for extensive discussions and Darrell Duffie and Thorsten Hens for valuable suggestions. Seminar participants at INSEAD and Zurich have provided valuable feedback. The authors gratefully acknowledge research support from the Swiss Finance Institute and from NCCR FINRISK of the Swiss National Science Foundation.

Abstract

We use equity options to examine how systematic and idiosyncratic variance risk are priced. The variances of both systematic and idiosyncratic stock returns comove countercyclically and command sizeable risk premia. Systematic variance risk exhibits a negative price of risk, whereas common idiosyncratic variance risk carries a large positive risk premium in the cross-section of options. This differential pricing of systematic and idiosyncratic variance risk explains several phenomena, (1) the relative prices of index and individual options, (2) the sizeable cross-sectional variation in stock option expensiveness, (3) the volatility mispricing puzzle documented by Goyal and Saretto (2009), and (4) the substantial returns earned on various option portfolio strategies. We find little evidence for ICAPM- and liquidity-based explanations of the observed patterns, but find support for theories of financial intermediation under capital constraints that account for the positive market price of idiosyncratic variance risk.

Systematic return (co)variances play a pivotal role in asset allocation and for the risk-return tradeoff in financial markets. There is now ample evidence that variances and correlations vary stochastically over time and exhibit several patterns. Andersen, Bollerslev, Diebold, and Ebens (2001) document that individual stock variances tend to move together and correlations are high when variance is high. Both variances and correlations tend to increase during crisis periods and when the stock market performs poorly.[1] As a result, states of the economy in which aggregate consumption is low and state prices are high coincide with high stock variances and correlations. Equities may thus offer fewer diversification benefits and less consumption insurance than suggested by looking at their unconditional moments. Augmenting investors' portfolios with traded instruments that allow hedging systematic variance risk therefore yields substantial welfare gains, and systematic variance risk carries a negative risk premium.[2] How idiosyncratic variance risk is priced in financial markets remains, however, largely an open question.

Equity options are the natural type of traded instruments to take positions on (co)variance risk. The market for equity options has been expanding dramatically over the past decades.[3] Equity options are now among the most important derivative securities and are used by institutional and individual investors for a variety of purposes, ranging from speculative trading on stock price movements to the transfer of (co)variance risk between investors. An improved understanding of the pricing of the various sources of risk in options markets and the benefits of diverse options strategies is essential for educating investors and informed policy-making.

Option prices exhibit a number of empirical regularities. First, as documented by Carr and Wu (2009) and Driessen, Maenhout, and Vilkov (2009), there are sizeable differences between the prices of index and stock options. Index options are "expensive," that is, they carry a large negative variance risk premium (meaning that the future variance implicit in index option prices

[1] In a seminal study, Black (1976) shows that volatility rises in falling financial markets. Longin and Solnik (2001) document that correlations rise in periods of high volatility. Erb, Harvey, and Viskanta (1994) show that stock market correlations tend to be higher when several countries are simultaneously in recession. Campbell, Lettau, Malkiel, and Xu (2001) document that idiosyncratic variances are countercyclical and positively related to market variance.

[2] See Driessen, Maenhout, and Vilkov (2009), Egloff, Leippold, and Wu (2009), DaFonseca, Grasselli, and Ielpo (2009), and Buraschi, Porchia, and Trojani (2010).

[3] Exchange-listed options began trading in the U.S. when the CBOE started on April 26, 1973. A total of 911 calls on 16 stocks were listed initially. Put trading was introduced in 1977. Options volume was 1.1 million contracts in the first year and exceeded 100 million contracts by 1981. Trading topped 1 (2) billion in 2004 (2006) and reached 3.59 billion contracts in 2009. See www.cboe.com for a historical digest.

exceeds the variance subsequently realized). By contrast, stock options tend to be "cheap," that is, the risk premium on stock variance is on average positive or close to zero, depending on the sample. Second, variance risk premia extracted from individual stock option prices exhibit sizeable cross-sectional variation along several dimensions. Specifically, controlling for exposure to market variance, variance risk premia for individual firms depend on firm characteristics such as size and the book-to-market ratio (Di Pietro and Vainberg (2006)). Third, as shown in Goyal and Saretto (2009), portfolios sorted on the ratio of past realized volatility to implied volatility earn abnormal returns, suggesting volatility mispricing in individual stock options. Fourth, as we document in the paper, a variety of portfolio sorts earn abnormal returns. Specifically, option returns are larger for stocks that have higher past realized variance, higher implied variance, higher past option returns, and higher exposure to index returns.

In this paper we establish a novel empirical regularity that allows reconciling these stylized facts. We show that common movements in the variances of idiosyncratic stock returns are priced in equity option returns. The market prices of the common idiosyncratic variance risk factors are strongly positive. In order to quantify these premia, we develop a parsimonious model of (co)variance swap pricing in the presence of stochastic systematic variances, idiosyncratic variances, and correlations. As suggested by Andersen et al. (2001), we build on a latent factor model of stock returns with stochastic variances, covariances, and correlations. In addition, we introduce common factors in the variances of idiosyncratic returns in order to capture the commonality in idiosyncratic variances observed empirically. This framework allows us to price variance swaps on return factor variances and on idiosyncratic variances. In our setting, the total variances of stock returns, return correlations across stocks, and stock market index variances are driven by three sources: (1) the variances of the return factors, (2) common factors in the variances of idiosyncratic returns, and (3) idiosyncratic movements in the variances of idiosyncratic returns. We estimate the risk premia associated with the various sources of variance risk by combining option and stock prices on several stock market indices with the corresponding data on their constituents.

Our empirical methodology relies on a simple identification strategy. Option prices can be used to compute synthetic variance swap rates, that is, to obtain the model-free no-arbitrage prices of forward contracts on the variance of the underlying.[4] In linear(ized) factor models of returns, the total variance (risk) is the sum of the systematic factor variances (risk) times the factor loadings and of the idiosyncratic variance (risk), and correlation risk is a composite of the two. The same relation holds under the physical and the risk-neutral measure (that is, for realized variances and for variance swap rates). In the data, we can measure total stock variance using stock returns and its pricing using variance swap rates obtained from option prices. Stock indices and index options are exposed only to systematic factor variance risk, allowing to identify factor variances and variance swap rates. Idiosyncratic variances and variance swap rates can then be backed out easily from the data for individual stocks. Using this approach, stock returns, return variances, variance swap rates, and variance risk premia can be decomposed into their corresponding factor and idiosyncratic components, allowing to separately measure the risk premia on systematic and idiosyncratic variance. To verify that our results are robust to the factor model assumptions, we establish the same pricing patterns in the returns on model-free dispersion trades, which are suitably constructed strategies of buying options on index constituents and selling index options with zero net exposure to certain shocks.

There is controversy in the empirical literature on the relative importance of variance and correlation risk and on the associated risk premia. On the one hand, Carr and Wu (2009) emphasize the importance of a *systematic variance risk factor* that carries a large negative risk premium, suggesting that "investors are willing to pay a premium to hedge away upward movements in the return variance of the stock market."[5] By contrast, Driessen et al. (2009) find that variance risk is not priced. They instead emphasize the importance of priced *correlation risk* as a separate source of risk that allows reconciling the presence of a large negative variance risk premium in S&P 100 index option prices with the absence of a variance risk premium in the prices of options on the index constituents.[6] These findings—only systematic variance risk is priced as opposed to correlation

[4]See Carr and Madan (1998) or Britten-Jones and Neuberger (2000) for a derivation of this no-arbitrage relation. Jiang and Tian (2005) show that the relation holds in the presence of jumps in the underlying asset price, and Carr and Wu (2009) show that the approximation error introduced by jumps is of third order.

[5]In more detail, Carr and Wu (2009) write: "The cross-sectional variation of the variance risk premiums possibly suggests that the market does not price all return variance risk in each stock, but only prices a systematic variance risk component in the stock market portfolio. [...] [W]e identify a systematic variance risk factor that the market prices heavily."

[6]In more detail, Driessen et al. (2009) write: "We demonstrate that priced correlation risk constitutes the missing link between unpriced individual variance risk and priced market variance risk, and enables us to offer a risk-based explanation for the discrepancy between index and individual option returns. Index options are expensive, unlike

risk is priced but variance risk is unpriced—are seemingly contradictory and, in addition, cannot account for a number of empirical facts. They can be reconciled, however, when considering the differences in sample selection across these papers.

In this paper we study a broad cross-section covering both S&P 100 and Nasdaq 100 stocks. We first establish that, while risk premia on total stock variance are zero on average for the S&P 100 stocks considered in Driessen et al. (2009), they are *positive* on average for Nasdaq 100 stocks. Second, consistent with Di Pietro and Vainberg (2006), stock variance risk premia depend on individual firm characteristics, suggesting that they reflect more than just exposure to systematic variance shocks. Third, we show that most of the movements in stock index variances (both S&P and Nasdaq) can be attributed to changes in the variances of the index constituents rather than to changes in return correlations. While both sources matter, the relationship between the average of constituent variances and index variance is much stronger than that between return correlations and index variance. Thus, the intuition that correlation risk should be priced and variance risk be unpriced (because shifts in index variance are driven by shifts in correlations and not by shifts in individual asset variances) may be misleading.[7]

We next quantify the variance risk premia on the common return factors and on assets' idiosyncratic return variances. Consistent with Carr and Wu (2009) who estimate a negative risk premium on systematic variance, we find the factor variance risk premia to be strongly negative. In our larger sample, however, idiosyncratic variance risk premia are strongly positive. In the cross-section of firms, the idiosyncratic variance risk premia increase with the market-to-book ratio, employee stock options, mutual fund ownership, and decrease with firm profitability and financial leverage. They are largely unrelated to bid-ask spreads in the option and in the underlying. We also document that common idiosyncratic variance risk factors are priced in the cross-section of equity option/variance swap returns. The bulk of the returns earned by the Goyal and Saretto (2009) strategy and on sort portfolios constructed on the basis of past realized variance, implied variance, past option returns, and exposure to index returns can be attributed to the portfolios' exposure to systematic and common idiosyncratic variance. Their abnormal returns are insignificant when one includes

individual options, because they allow investors to hedge against positive market-wide correlation shocks and the ensuing loss in diversification benefits."

[7] We formally establish the relationship between variance and correlation risk premia in the paper.

risk factors for systematic and common idiosyncratic variances in the pricing equation. Thus, our results demonstrate that equity options have returns that are not spanned by the Fama-French and momentum factors. When assessing the profitability of option strategies, it is important to include both systematic and idiosyncratic variance risk factors.

We investigate a number of potential explanations for the sign and size of systematic and idiosyncratic variance risk premia. We first consider Merton's (1973) ICAPM and investigate whether systematic and idiosyncratic variances predict the future state of the economy. We confirm that systematic variance negatively predicts GDP and investment growth. However, we find no support that idiosyncratic variances have predictive power (Campbell et al. (2001)). Using Campbell's (1993) result that variables whose innovations are associated with good (bad) news about future investment opportunities have a positive (negative) risk price, we also investigate the relationship between market returns, systematic variance, and idiosyncratic variances. We find that systematic and idiosyncratic variances are unrelated to future market returns, that systematic variance is positively related to future market variance, and that idiosyncratic variance is unrelated to future market variance. Thus, increases in systematic variance are bad news about future investment opportunities, while increases in idiosyncratic variance have no obvious macroeconomic relevance. Our conclusion is that the ICAPM can account for the negative risk premium on systematic variance but not for the positive risk premium on idiosyncratic variance.

We then consider explanations based on financial market imperfections. However, we find little support for an illiquidity-based explanation of the observed patterns. In particular, idiosyncratic variance risk premia are largely unrelated to the bid-ask spread on the option and the underlying—contrary to what one would expect if the positive idiosyncratic variance risk premium reflects compensation for the illiquidity of stock options and the costs associated with hedging them. Last, we explore the role of capital-constrained financial intermediaries for risk compensation in the options market and offer agency-based explanations for the puzzling sign of idiosyncratic variance risk premia. Financial intermediaries play a pivotal role as counterparties in the options market. They provide liquidity to hedgers and speculators and absorb much of the trading. Many investor groups have a preference for negative skewness and, therefore, supply individual stock options. For instance, a prominent hedge fund strategy is to short individual stock variance, generating a high propensity of small gains and infrequent large losses ("picking up nickels in front of a steamroller"). We capture several sources of supply by measuring option writing by investment funds to enhance yields and attract fund flows (Malliaris and Yan (2010)) and by holders of firm-issued options such as employee stock options to hedge the convexity in their payoffs.

We develop a simple model of option market-making to rationalize the estimated risk premia and test the hypotheses structurally. The model captures that idiosyncratic movements in the variances of idiosyncratic returns are diversified away in a large portfolio of options. What remains is the risk that the variances of idiosyncratic returns move in a systematic way. Intermediaries, hence, cannot hedge options perfectly and, as a result, are sensitive to risk. To the extent that investors are net suppliers of individual stock options (Garleanu, Pedersen, and Poteshman (2009)), common idiosyncratic variance risk commands a positive risk premium in equilibrium. In addition, the equilibrium pricing condition yields that (1) the cross-section of variance risk premia reflects the asset's exposure to the common idiosyncratic variance factor(s), the price of risk for common idiosyncratic variance is larger at times (2) when the total net supply of stock options is larger and (3) when the riskiness of common idiosyncratic variance is larger, and an asset's variance risk premium is higher (4) the larger the net supply of variance for that asset and (5) the more variable the asset's idiosyncratic variance. Consistent with this hypothesis, we find that idiosyncratic variance risk premia are higher, the greater the number of firm-issued options outstanding and the larger mutual fund ownership. Overall, we find empirical support for four of the five predictions.

The remainder of the paper is organized as follows. Section 1 describes the factor model of returns underlying our analysis, its implications for variance swap pricing, and establishes the relationship between variance and correlation risk premia. Section 2 describes the data and shows how to extract factor and idiosyncratic variance swap rates. Section 3 provides descriptive statistics on variance risk premia and conducts a specification analysis of the model. Section 4 computes factor and idiosyncratic variance risk premia and documents that both systematic and common idiosyncratic variance risk are priced in the cross-section of equity option returns. Section 5 investigates potential explanations for the difference in the signs of these premia and the cross-sectional determinants of idiosyncratic variance risk premia. Section 6 describes the robustness checks we have performed. Section 7 concludes. Technical developments are gathered in the Appendix.

1 A Model of (Co)variance Swap Pricing under Systematic and Idiosyncratic Variance Risk

In this section we develop a financial market model that yields tractable variance swap pricing formulas when asset returns and return (co)variances are allowed to follow a general factor structure. Section 1.1 describes our assumptions on asset returns and variance dynamics. Section 1.2 discusses the arbitrage-free pricing of (co)variance swaps. Section 1.3 quantifies variance risk premia and establishes the relationship between variance and correlation risk premia.

1.1 The model

We consider an economy with N risky assets indexed by $n = 1, \ldots, N$. Asset prices $S_{n,t}$ are driven by J systematic return factors F_t and an idiosyncratic component. The instantaneous excess return on risky asset n under the risk-neutral measure Q is given by

$$\frac{dS_{n,t}}{S_{n,t}} - r_{f,t}dt = \underbrace{\beta'_{n,t}dF_t^Q}_{\text{Systematic return}} + \underbrace{\sqrt{V_{n,\epsilon,t}}dZ_{n,\epsilon,t}^Q}_{\text{Idiosyncratic return}} , \tag{1}$$

where $r_{f,t}$ denotes the riskless interest rate, $\beta_{n,t}$ the J-dimensional vector of asset n's factor exposures, and the last term captures the asset's idiosyncratic return. The term $V_{n,\epsilon,t}$ measures the instantaneous variance of asset n's idiosyncratic return component and $Z_{n,\epsilon,t}^Q$ is a standard Brownian motion. The vector of instantaneous factor returns dF_t^Q is assumed to have risk-neutral dynamics

$$dF_t^Q = \Sigma_t^{1/2} dZ_t^Q , \tag{2}$$

where Σ_t denotes the instantaneous factor variance-covariance matrix and Z_t^Q is a standard Brownian motion vector. As is standard in factor models, we assume that all co-movements in returns are caused by exposure to the J common factors, so that $dZ_{m,\epsilon,t}^Q dZ_{n,\epsilon,t}^Q = 0$ for all $m \neq n$, and that the idiosyncratic returns are independent of the factor returns, $dZ_t^Q dZ_{n,\epsilon,t}^Q = 0$ for all $n = 1, \ldots, N$.

We allow the instantaneous factor variance-covariance matrix Σ_t and the instantaneous variances of the assets' idiosyncratic returns, $V_{n,\epsilon,t}$, to follow stochastic processes that are correlated with each other and associated with risk premia. Specifically, given the evidence in Campbell et al. (2001) that assets' idiosyncratic return variance is time varying and related to market variance, we allow the idiosyncratic return variances $V_{n,\epsilon,t}$ to follow a factor structure:

$$V_{n,\epsilon,t} = \underbrace{\gamma'_{n,t}\Gamma_t}_{\text{Common idiosyncratic variance}} + \underbrace{\tilde{V}_{n,\epsilon,t}}_{\text{Truly idiosyncratic variance}} , \tag{3}$$

where Γ_t is a G-dimensional stochastic process of common idiosyncratic variance factors that may be correlated with Σ_t, $\gamma_{n,t}$ is a G-dimensional vector of factor exposures, and $\tilde{V}_{n,\epsilon,t}$ denotes the part of asset n's idiosyncratic return variance that is specific to asset n; we will call $\tilde{V}_{n,\epsilon,t}$ asset n's "truly idiosyncratic" variance.

Differences in exposure to systematic and idiosyncratic variance risk between individual stock and index variances are key to our empirical identification strategy. In addition to the n individual assets, consider the pricing of asset portfolios or stock market indices, indexed by $p = 1, \ldots, P$. Let $I_{p,t}$ denote the price of index p at time t, $a_{n,p}$ be the number of shares of asset n in index p and $w_{n,p,t} = a_{n,p}S_{n,t}/I_{p,t}$ asset n's weight in the index at time t. The price process of stock market index p satisfies $I_{p,t} = \sum_{n=1}^{N} a_{n,p}S_{n,t}$ with dynamics

$$dI_{p,t} = \sum_{n=1}^{N} a_{n,p}dS_{n,t} = I_{p,t}(r_t dt + \beta'_{I,p,t}dF_t^Q + \sum_{n=1}^{N} w_{n,p,t}\sqrt{V_{n,\epsilon,t}}dZ_{n,\epsilon,t}^Q) , \tag{4}$$

where $\beta_{I,p,t} = \sum_{n=1}^{N} w_{n,p,t}\beta_{n,t}$ denotes the weighted-average exposure of the index constituents to the return factors.[8] Under the above assumptions, the (instantaneous) variance of asset returns $dS_{n,t}/S_{n,t}$ and of index returns $dI_{p,t}/I_{p,t}$ are given by

$$\sigma_{n,t}^2 = \beta'_{n,t}\Sigma_t\beta_{n,t} + V_{n,\epsilon,t} , \tag{5}$$

$$\sigma_{I,p,t}^2 = \beta'_{I,p,t}\Sigma_t\beta_{I,p,t} + \sum_{n=1}^{N} w_{n,p,t}^2 V_{n,\epsilon,t} . \tag{6}$$

The first term in expression (6) captures how the factor (co)variances affect index variance, and the second term is negligible provided that the index is well-balanced. Hence, index variance is driven

[8] The last term in equation (4) will be close to zero as a result of diversification so long as the index is well-balanced. Our empirical methodology is robust to under-diversification of the index.

largely by factor variances while stock variances depend on factor variances and the variance of the idiosyncratic return component. Next we describe the pricing of variance contracts in this setting.

1.2 The pricing of (co)variance contracts

Variance swaps are contracts that at maturity pay the realized variance, RV, over a fixed horizon net of a premium called the *variance swap rate*, VS. The latter is set by the contracting parties such that the variance swap has zero net market value at entry. The advantage of the factor model of returns laid out in the previous section is that it yields a simple characterization of variance swap rates on individual assets and asset portfolios.

Denote by $VS_{n,t,\tau}$ and $VS_{n,\epsilon,t,\tau}$ the arbitrage-free variance swap rates on the total and, respectively, idiosyncratic return of asset $n = 1, \ldots, N$ at time t with maturity $t+\tau$. That is, $VS_{n,\epsilon,t,\tau}$ is the variance swap rate of a synthetic asset exposed only to asset n's idiosyncratic risk. Similarly, denote by $VS_{t,\tau}$ the matrix of arbitrage-free (co)variance swap rates on the systematic return factors at time t with maturity $t+\tau$. That is, $VS^{jj}_{t,\tau}$ is the variance swap rate of a synthetic asset with unit exposure to factor j, zero exposure to all other factors, and no idiosyncratic risk, and $VS^{ij}_{t,\tau}$ is the covariance swap rate between factors i and j. By absence of arbitrage, one has

$$
\begin{array}{ll}
\text{Total variance swaps:} & VS_{n,t,\tau} = \frac{1}{\tau} E^Q_t[\int_t^{t+\tau} \sigma^2_{n,u} du] \\
\text{Factor (co)variance swaps:} & VS_{t,\tau} = \frac{1}{\tau} E^Q_t[\int_t^{t+\tau} \Sigma_u du] \\
\text{Idiosyncratic variance swaps:} & VS_{n,\epsilon,t,\tau} = \frac{1}{\tau} E^Q_t[\int_t^{t+\tau} V_{n,\epsilon,u} du]
\end{array}
$$

Variance swap rates follow a linear factor structure when asset returns are driven by common factors. For ease of exposition, in this section we present the characterization for the case where factor exposures over the life of the contract are known and constant, i.e., $\beta_{n,u} = \beta_{n,t}$ over $u \in [t, t+\tau)$. In this case, the variance swap rate on asset n can be decomposed into a factor component—driven by the asset's factor exposures $\beta_{n,t}$ and the (co)variance swap rates $VS_{t,\tau}$ of the return factors—and into the variance swap rate on asset n's idiosyncratic return, $VS_{\epsilon,n,t,\tau}$:

$$VS_{n,t,\tau} = \beta'_{n,t} VS_{t,\tau} \beta_{n,t} + VS_{n,\epsilon,t,\tau} \ . \tag{7}$$

Similarly, assuming constant index weights $w_{n,p,u} = w_{n,p,t}$ over $u \in [t, t+\tau)$, $\beta_{I,p,u} = \beta_{I,p,t}$ over $u \in [t, t+\tau)$ and the variance swap rate on index p can be written as:

$$VS_{I,p,t,\tau} = \frac{1}{\tau} E_t^Q [\int_t^{t+\tau} \sigma_{I,p,u}^2 du] = \beta'_{I,p,t} VS_{t,\tau} \beta_{I,p,t} + \underbrace{\sum_{n=1}^{N} w_{n,p,t}^2 VS_{n,\epsilon,t,\tau}}_{\approx 0} \ . \tag{8}$$

Appendix A contains details on how this characterization (and consecutive results) have to be modified when factor exposures over the life of the variance swap are time-varying and/or parameter uncertainty is present.[9] Appendix B contains simulation evidence on the accuracy of approximation (8). To assess this accuracy, we compare the variance swap rates (8) with those obtained by simulating the stochastic processes and accounting for the random variation in weights. We find the approximation to be very accurate. The approximation exhibits a small downward bias for low initial variances and a small upward bias for large initial variances. Even in the worst case simulation scenario the approximation error remains below two per cent.

1.3 Variance and correlation risk premia

The financial market model laid out in Section 1.1 yields tractable expressions for the variance risk premia on individual stocks and indices. Following Driessen et al. (2009), the instantaneous variance risk premium on asset n is given by $VRP_{n,t} \equiv E_t^Q[d\sigma_{n,t}^2] - E_t^P[d\sigma_{n,t}^2]$ where Q denotes the risk-neutral measure and P the physical probability measure. Similarly, the variance risk premium on the return factors is $VRP_t \equiv E_t^Q[d\Sigma_t] - E_t^P[d\Sigma_t]$ and that on asset n's idiosyncratic return component $VRP_{n,\epsilon,t} \equiv E_t^Q[dV_{n,\epsilon,t}] - E_t^P[dV_{n,\epsilon,t}]$. Using expression (6) and assuming deterministic factor exposures or, alternatively, no risk premium on changes in factor exposures, the variance risk premium on asset $n = 1, \ldots, N$ equals

$$VRP_{n,t} = \beta'_{n,t} VRP_t \beta_{n,t} + VRP_{n,\epsilon,t} \ . \tag{9}$$

[9] As we show in Appendix A, sufficient conditions for our analysis to go through are that individual assets' factor exposures are martingales and changes in factor exposures are uncorrelated with factor variances and covariances Σ_t. We also show empirically in Section 4.1 that accounting for time variation in factor exposures does not substantively affect our results.

Hence, variance risk premia inherit the linear factor structure. Expression (9) states that the variance risk premium on an individual asset is given by the sum of the variance risk premium arising from the asset's exposure to the common return factors (first term) and the variance risk premium on the idiosyncratic return (second term). The first term is itself driven by the (co)variance risk premia on the common return factors. Similarly, using (6) the variance risk premium on index p, $VRP_{I,p,t} \equiv E_t^Q[d\sigma_{I,p,t}^2] - E_t^P[d\sigma_{I,p,t}^2]$, is given by:

$$VRP_{I,p,t} = \beta'_{I,p,t} VRP_t \beta_{I,p,t} + \underbrace{\sum_{n=1}^{N} w_{n,p,t}^2 VRP_{n,\epsilon,t}}_{\approx 0} \, . \tag{10}$$

The variance risk premium on stock index p is given predominantly by the variance risk premium arising from the index's exposure to the common return factors. The second term in the sum is the variance risk premium on the idiosyncratic return of the index constituents, weighted by the squared index weights. Its contribution is negligible so long as the index is well-balanced.

What is the relationship between variance and correlation risk premia? Letting $\rho_{m,n,t} = \frac{\beta'_{m,t}\Sigma_t\beta_{n,t}}{\sigma_{m,t}\sigma_{n,t}}$ denote the instantaneous return correlation between assets m and n, Itô's lemma reveals that correlation risk premia are driven by the (co)variance risk premia on the common return factors, VRP_t, and the variance risk premia of the individual assets, $VRP_{n,t}$:

$$E_t^Q[d\rho_{m,n,t}] - E_t^P[d\rho_{m,n,t}] \quad = \quad \frac{1}{\sigma_{m,t}\sigma_{n,t}} \beta'_{m,t} VRP_t \beta_{n,t} - \frac{\rho_{m,n,t}}{2}\left(\frac{VRP_{m,t}}{\sigma_{m,t}^2} + \frac{VRP_{n,t}}{\sigma_{n,t}^2}\right) \, . \tag{11}$$

From (9), the variance risk premia of the individual assets themselves depend on the (co)variance risk premia on the common return factors and on the variance risk premia on assets' idiosyncratic returns. Thus, correlation risk premia are ultimately a combination of the (co)variance risk premia on the common return factors and the variance risk premia on assets' idiosyncratic returns.[10]

[10] Hence, by explicitly accounting for the factor structure of asset returns, (9) and (10) allow identifying the (co)variance risk premia on the common return factors and the variance risk premia on assets' idiosyncratic returns separately. By contrast, the correlation risk premia estimated in, for instance, Driessen et al. (2009) identify a combination of the two components. In Appendix C we show how to reconcile expression (10) with the expression for the index variance risk premium derived by Driessen et al. (2009). The relationship between variance and correlation risk premia becomes particularly intuitive when the common return factors are uncorrelated. In this case, correlation risk premia among assets can only arise if variance risk (either factor variance risk, or idiosyncratic variance risk, or both) are priced. Specifically, the first term in (11) becomes $\frac{1}{\sigma_{m,t}\sigma_{n,t}} \sum_{j=1}^{J} \beta_{m,t}(j)\beta_{n,t}(j)(E_t^Q[d\Sigma_t^{jj}] - E_t^P[d\Sigma_t^{jj}])$, where $\beta_{m,t}(j)$ denotes the jth component of $\beta_{m,t}$. Thus, correlation risk premia $E_t^Q[d\rho_{m,n,t}] - E_t^P[d\rho_{m,n,t}]$ are driven

2 Data and Empirical Methodology

The data for our empirical analysis consists of option price data from OptionMetrics and daily index and constituent stock returns from CRSP. We also employ data from Compustat, Thomson Financial, and other sources. The sample period ranges from January 2, 1996 to October 31, 2009. For most of the analysis, we use data on two stock market indices, the S&P 100 index (OEX) and the Nasdaq 100 index (NDX), for which liquid options are available throughout the sample, and on all their constituent stocks.[11] For both indices, we obtain historical index weights of the constituent stocks on each trading day in the sample period as described in Appendix D.

2.1 Synthesizing variance swap rates

Variance swaps are traded over-the-counter and swap quotes are difficult to obtain at low cost. One can, however, easily compute *synthetic* variance swap rates from option prices using the methodology outlined in Demeterfi, Derman, Kamal, and Zou (1999) and Carr and Wu (2009).

For both indices and the 452 stocks that were members of one of the two indices at some point during our sample period, we extract daily put and call option implied volatilities for a constant maturity of one month (30 calendar days) from the OptionMetrics database. The OptionMetrics volatility surface file provides option implied volatilities for deltas between 0.2 and 0.8 in absolute value in steps of 0.05. The data are adjusted for early exercise. On the basis of these implied volatilities, we compute variance swap rates using the methodology described by Carr and Wu (2009). We linearly interpolate the volatility surface between the points provided in the OptionMetrics database using log moneyness $k \equiv \ln(K/F)$, where K is the strike price and F the futures price, to obtain the Black-Scholes implied volatility for moneyness level k, $\sigma(k)$. We then use these implied volatilities to evaluate the cost of the replicating portfolio of a variance swap with maturity

only by the variance risk premia on the common return factors, $E_t^Q[d\Sigma_t^{jj}] - E_t^P[d\Sigma_t^{jj}]$, and the variance risk premia of the individual assets, $VRP_{n,t}$.

[11] We also considered the Dow Jones Industrial Average index (DJX), for which options are available since September 24, 1997. However, DJX's exposure to the common return factors is very similar to that of the S&P 100 index. As a result, the DJX index does not add sufficient information to identify the variance swap rates and variance risk premia on the common return factors, and one runs into multicollinearity problems. The same issue arises with the S&P 500 index (SPX), for which options are available for the entire sample.

$t + \tau$ as

$$VS_{t,\tau} = \frac{2}{\tau}[\int_{-\infty}^{0} \left(-e^{-k}N(-d_1(k)) + N(-d_2(k))\right) dk + \int_{0}^{\infty} \left(e^{-k}N(d_1(k)) - N(d_2(k))\right) dk] \ , \tag{12}$$

where $N(\cdot)$ denotes the standard normal cumulative distribution function and

$$d_1(k) = \frac{-k + \sigma^2(k)\tau/2}{\sigma(k)\sqrt{\tau}} \ , \quad d_2(k) = d_1(k) - \sigma(k)\sqrt{\tau}. \tag{13}$$

2.2 Identifying factor and idiosyncratic variance swap rates and risk premia

Variance swap rates on return factor variance, $VS_{t,\tau}$, and on idiosyncratic variances, $VS_{n,\epsilon,t,\tau}$, are not readily available in financial markets. They can be synthesized, however, so long as variance swap rates on individual stocks and on a sufficient number of stock indices are available (which, in turn, can be replicated using static portfolios of out-of-the-money call and put options and a delta-hedging strategy in the underlying stock). This construction is possible since variance swap rates on individual assets and on stock indices are linked by absence of arbitrage to idiosyncratic and factor variance swap rates, but with different weights, and these weights are known functions of the index constituents' exposures to the return factors and their weights in the index.

In the following, we show how to separately identify the factor and idiosyncratic components of variance swap rates by combining information on individual assets' and indices' variance swap rates. We present results for the case of constant and known factor exposures. Appendix A shows that our methodology can be applied with minor modifications to situations with time-varying factor exposures and parameter uncertainty.

One can construct *adjusted* index variance swap rates $y_{p,t,\tau}$, $p = 1, \ldots, P$, that are robust to under-diversification in the index (i.e., neutral to idiosyncratic variances) by combining individual variance swap rates $VS_{n,t,\tau}$ and index variance swap rates $VS_{I,p,t,\tau}$ as follows:

$$y_{p,t,\tau} \equiv VS_{I,p,t,\tau} - \sum_{n=1}^{N} w_{n,p,t}^2 VS_{n,t,\tau} = \beta'_{I,p,t} VS_{t,\tau} \beta_{I,p,t} - \sum_{n=1}^{N} w_{n,p,t}^2 \beta'_{n,t} VS_{t,\tau} \beta_{n,t} + \epsilon_{p,t} \ , \tag{14}$$

where $\epsilon_{p,t}$ denotes an error term that reflects the approximation error in (8) and measurement error in the data. Combining the expressions for all indices yields a linear system in $y_{t,\tau} = (y_{1,t,\tau}, \ldots, y_{P,t,\tau})'$:

$$y_{t,\tau} = X_t \Phi_{t,\tau} + \epsilon_t, \tag{15}$$

where

$$X_t = \begin{pmatrix} A_{1,1,t} & \ldots & A_{1,J,t} & B_{1,1,2,t} & \ldots & B_{1,J-1,J,t} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ A_{P,1,t} & \ldots & A_{P,J,t} & B_{P,1,2,t} & \ldots & B_{P,J-1,J,t} \end{pmatrix}, \tag{16}$$

$$\Phi_{t,\tau} = \begin{pmatrix} VS^{11}_{t,\tau} & \ldots & VS^{JJ}_{t,\tau} & VS^{12}_{t,\tau} & \ldots & VS^{J-1,J}_{t,\tau} \end{pmatrix}', \tag{17}$$

and (with i and j in parenthesis denoting the vector component),

$$A_{p,j,t} \equiv [\beta_{I,p,t}(j)]^2 - \sum_{n=1}^{N} w^2_{n,p,t} \beta_{n,t}(j)^2, \quad j = 1, \ldots, J \tag{18}$$

$$B_{p,i,j,t} \equiv 2[\beta_{I,p,t}(i)\beta_{I,p,t}(j) - \sum_{n=1}^{N} w^2_{n,p,t} \beta_{n,t}(i)\beta_{n,t}(j)], \quad i = 1, \ldots, J-1, \; j = i+1, \ldots, J. \tag{19}$$

Using expression (15) as the measurement equation, the linear Kalman filter consistently extracts the factor (co)variance swap rates $\Phi_{t,\tau}$ from the adjusted index variance swap rates y_t and the matrix of adjusted factor exposures X_t. Once the factor variance swap rates are known, the idiosyncratic variance swap rates are simply given by $VS_{n,\epsilon,t,\tau} = VS_{n,t,\tau} - \beta'_{n,t} VS_{t,\tau} \beta_{n,t}$.

The methodology outlined above allows decomposing variance swap rates and variance swap returns into their factor and idiosyncratic components. With a panel data set of variance swap returns constructed in this way at hand, we can study the risk factors affecting the cross-section of variance swap returns. Before doing so in Section 4, we present some descriptive statistics in the next section. Throughout the empirical analysis, we use variance swap rates and variance risk premia over one-month horizons and compute realized variances using rolling one-month (21 trading day) windows. Whenever we refer to an average quantity for index constituents, the average is computed using the constituent stocks' weight in the index.

3 Variance Dynamics and Cross-Sectional Pricing of Variance Risk

This section provides descriptive statistics on stock and index return variances, variance swap rates, and variance risk premia. We start by characterizing the properties of individual stock and index variances in Section 3.1. In Section 3.2, we quantify variance risk premia on the S&P 100 and Nasdaq 100 indices and contrast them with the variance risk premia on the index constituents. We also establish several striking regularities in the cross-section of stock variance risk premia. In Section 3.3, we perform a specification analysis of the factor model laid out in Section 1.1.

3.1 The time-series of individual stock and index variances

Figure 1 illustrates the behavior of individual and index variances over time. The figure plots the realized variance of index returns, the average realized variance of the index constituents, the average correlation between index constituents, and the product of the average realized variance and the average correlation for the S&P 100 and Nasdaq 100 indices from January 1996 to October 2009. The top panel depicts the four series for the S&P 100 index, while the bottom panel reports them for the Nasdaq 100 index.

[**Figure 1 about here**]

Figure 1 highlights a number of empirical patterns in stock variances and correlations that have been documented in the literature. Individual stock variances, index variances, and return correlations all comove. They increase during crisis periods and when the stock market performs poorly ("leverage effect", Black (1976)). There exists commonality in variances, that is, individual stock variances tend to move together (Andersen et al. (2001)). Correlations are high when variances are high (Longin and Solnik (2001)). Further, volatility exhibits spikes and mean-reverts, consistent with the findings in the GARCH literature. Last, the time-variation in the difference between average constituent variances and index variances shows that idiosyncratic variances are highly time-varying, countercyclical, and positively related to market variance (Campbell et al. (2001)).

The figure also reveals that index variance is driven both by constituent stock variances and by their correlations—as predicted by the model in Section 1.1. The strong co-movements between individual asset variances and index variances apparent in Figure 1 reflect the fact that both are exposed to shifts in the variances of the common return factors (the same holds for variance swap rates and variance risk premia). By contrast, Driessen et al. (2009) offer an alternative channel and emphasize the importance of *correlation risk* in reconciling the large negative variance risk premium in S&P 100 index option prices with the absence of a variance risk premium in the option prices of index constituents. They develop the intuition that increases in market variance are driven primarily by increases in return correlations, rather than by market-wide movements in individual variance.[12] For this intuitive argument to be empirically relevant, one would expect a strong relation between index variance and constituent correlations.

Table 1 reports the correlations between the different series depicted in Figure 1 and conveys the same message. Both the average constituent variance and the average constituent correlation are correlated with index variance, but neither is close to being perfectly correlated with index variance. For both indices, the correlation between index variance and average constituent variance is significantly higher than that between index variance and average constituent correlation, and the correlation between index variance and the product of average constituent variance and average constituent correlation is near perfect. These results demonstrate that in order to accurately capture changes in index variance, one needs to account for both changes in constituent asset variances and changes in their correlations.

[**Table 1 about here**]

Figure 2 plots variance swap rates on the index against the average rate of the index constituents for the S&P 100 and Nasdaq 100 indices. As for realized variances depicted in Figure 1, asset and index variance swap rates strongly co-move, but the correlation is imperfect. Regressing the S&P 100 and Nasdaq 100 index variance swap rates on the average variance swap rate of their constituents yields R^2 values of 79.47% and 93.88%, respectively.[13]

[12]Driessen et al. (2009) explain that "a market-wide increase in correlations negatively affects investor welfare by lowering diversification benefits and by increasing market volatility, so that states of nature with unusually high correlations may be expensive. [...] We demonstrate that priced correlation risk constitutes the missing link between unpriced individual variance risk and priced market variance risk, and enables us to offer a risk-based explanation for the discrepancy between index and individual option returns."

[13]Performing the same analysis on variance swap returns produces similar results: regressing the variance swap return of the S&P 100 and Nasdaq 100 index on the average variance swap return of their constituents yields R^2 values of 74.53% and 78.38%, respectively.

[**Figure 2 about here**]

3.2 The cross-section of variance risk premia

As little is known about the pricing of variance risk in the cross-section, it is useful to start by comparing variance risk premia on different stocks and indices. Following Carr and Wu (2009), we measure variance risk premia using the average returns on short-dated variance swaps, computed as holding period returns from a long variance swap position over the period t to $t+\tau$ (τ is chosen to be one month):

$$r_{n,t,\tau} = \frac{RV_{n,t,\tau} - VS_{n,t,\tau}}{VS_{n,t,\tau}} \ , \tag{20}$$

where $RV_{n,t,\tau} = \frac{1}{\tau}\int_t^{t+\tau} \sigma_{n,u}^2 du$ denotes the realized variance and $VS_{n,t,\tau}$ the variance swap rate on the stock or, respectively, index. Consistent with the prior literature (Carr and Wu, 2009, Driessen et al., 2009), in our sample the variance risk premia on the S&P 100 and Nasdaq 100 indices are strongly negative, with average values of -15.11% and -5.36% per month, respectively (the Newey-West t-statistics with 20 lags are -3.13 and -1.27). Also consistent with Driessen et al. (2009), the average variance swap returns on the S&P 100 constituents are marginally positive but statistically insignificant, with a value of 3.70% per month (Newey-West t-statistic 1.06). By contrast, we find the average variance swap returns on the Nasdaq 100 index constituents to be economically and statistically positive, with a value of 9.64% per month (Newey-West t-statistic 3.21). Hence, the stylized fact of a zero variance risk premium on individual stock variances in Driessen et al. (2009) does not generalize to Nasdaq 100 stocks, emphasizing the need to investigate factor and idiosyncratic components of assets' variance risk premia separately.

There are also sizeable differences in variance risk premia in the cross-section. Table 2 reports the monthly returns on equally-weighted sort portfolios of single-stock variance swaps constructed at the end of each month based on the ratio of historical variance during the previous month to the variance swap rate at the end of the month (Panel A), variance swap returns in the previous month (Panel B), individual stocks' historical variance during the previous month (Panel C), variance swap rates at the end of the month (Panel D), and the underlying stocks' exposure to S&P 100 and Nasdaq 100 index returns computed by OLS (Panels E and F). In order to avoid survival bias issues, only those stocks that are members of the S&P 100 or Nasdaq 100 index as of the portfolio formation date are considered in the analysis. The results including all the stocks in our sample are similar. Sharpe ratios are expressed in annual terms. In each panel, we also report the average monthly turnover of each portfolio, computed as the average fraction of stocks that are included in the portfolio in a given month but were not in that portfolio in the previous month. Row 1 (5) reports the return of the portfolio containing stocks with the lowest (highest) values of the sort variable, and row "5 − 1" that of a long-short portfolio.

[**Table 2 about here**]

The strategy in Panel A of Table 2 is similar to Goyal and Saretto (2009, *GS* henceforth). *GS* find that a trading strategy that is long (short) options with a large (small) ratio of realized volatility during the previous twelve months to at-the-money implied volatility as of the portfolio formation date earns large abnormal returns. In Panel A, we report monthly returns for the strategy used in *GS*, except that we compute realized variance over the previous month rather than over the previous 12 months and use the variance swap rate instead of at-the-money implied volatility.[14] A similar Sharpe ratio is achieved for portfolios based on the lagged variance swap return (Panel B). Thus, variance swap returns on individual stocks are highly persistent. However, the portfolio's high turnover of 72.76% per month suggests that this persistence may be short-lived.

Panels C and D of Table 2 report the returns of portfolios constructed using the components of the *GS* ratio. Panel C reveals that variance swap returns are significantly higher for stocks that had high realized variance in the previous month. A long-short portfolio of variance swaps constructed on the basis of historical variance yields a monthly average return of 15.81%, with a Sharpe ratio of 1.55. Average portfolio turnover is much lower at 43.56% per month, suggesting a risk-based rather than a mispricing explanation for the return differences. Panel D reveals that similar returns are achieved when the portfolios are constructed on the basis of the variance swap rate. Average turnover is about half that of the historical variance strategy in Panel C, reflecting that variance swap rates are more persistent than historical variance.

[14] The strategy in Panel B of Table 2 is also similar to *GS*, except that sorting on the variance swap return in the previous month amounts to sorting on the ratio of historical volatility in the prior month to implied volatility in the previous month, as opposed to the ratio of realized volatility in the prior month to implied volatility at the end of the month as in *GS*.

Panels E and F of Table 2 focus on a different return pattern. In Panels E and F we sort variance swaps on stock return betas. If only systematic variance risk is priced and commands a negative risk premium, high-beta stocks should exhibit lower variance risk premia. As can be seen, however, variance swap returns are higher for high-beta stocks, irrespective of whether beta is computed with respect to the S&P 100 or the Nasdaq 100 index. Hence, past variance swap rates and the volatility mispricing documented by *GS* are not the only determinants of expected variance swap returns. Importantly, the portfolios formed by sorting on beta differ in terms of turnover from the other sorts. The portfolios formed on the basis of index exposure have average monthly turnover of less than 2%, suggesting that different risk exposures may drive the portfolio returns and not mispricing.

3.3 Specification analysis

Before turning to a discussion of the variance risk decomposition, we apply a number of criteria to verify that the specification in Section 1 is empirically valid. We check, first, that the latent return factors reproduce the time-series of index returns and, second, that the factor variances capture realized index variances and their dynamics. To test the latter requirement, we use the fact that residual returns are correlated when the number of return factors is too small. Third, we verify that the return factors can account for the correlations in individual asset returns and their time-variation. For this to be the case, correlations in residual returns should be close to zero at all times. Appendix E details our return factor extraction methodology and confirms that our empirical model with two common return factors satisfies all three criteria.

Another central prediction of the factor model laid out in Section 1 is that individual asset variances, variance swap rates, and variance risk premia inherit a factor structure. To investigate whether this prediction is empirically valid, we perform separate factor analyses of the panels of realized variances, variance swap rates, and variance swap returns.[15] Table 3 summarizes the

[15] To our knowledge, the factor structure of individual assets' variance swap rates has not been documented previously. Carr and Wu (2009) compute variance betas for each of the stocks they consider by regressing these stocks' realized variances on the realized variance of the S&P 500 index, which they take as a proxy for the market portfolio variance. They document that stocks with higher variance betas have variance risk premia that are more strongly negative. However, they do not investigate co-movements in asset variance swap rates. Vilkov (2008) investigates the factor structure of variance swap returns, but not that of variance swap rates.

results.

[Table 3 about here]

Table 3 reveals, first, the presence of strong co-movements in variances, variance swap rates, and variance swap returns and, second, common variance movements in the cross-section that are absent from the index. In particular, in individual assets' realized variances a single factor explains 44% of the variation—as can be seen in the first row of Panel A. Two (four) factors explain up to 55% (62%). The first row of Panel B reveals that commonality is even stronger for variance swap rates than for realized variances, suggesting that shocks to "truly idiosyncratic" variance are short-lived and only marginally alter (risk neutral) expectations of future variance. A single factor explains around 56% of the variation in individual assets' variance swap rates, and two (four) factors account for about 70% (78%) of the variation. To complete the analysis, the results in the first row of Panel C show that common factors are also present in individual assets' variance swap returns. Not surprisingly, idiosyncratic variation in variance swap returns is stronger than in realized variances and variance swap rates. A single factor explains about 26%, and four factors explain about 32% of the variation in variance swap returns. These values are comparable to those for stock returns.

In summary, increases in market variance are closely tied to movements in constituent variances and are best understood by studying the time-series behavior of the variance of common return factors and common movements in the variances of idiosyncratic returns, both of which cause a market-wide increase in the return variance of individual names.

4 The Pricing of Systematic and Idiosyncratic Variance Risk

In this section, we characterize the properties of variance swap rates on the common return factors and on idiosyncratic returns and quantify the associated variance risk premia. We also provide evidence that common idiosyncratic variance risk is priced in the cross-section of equity options. We extract the factor variance swap rates using both constant and, for robustness, time-varying factor exposures for individual assets. The results for both settings are similar throughout.

4.1 Variance swap rates on factor and idiosyncratic variances

Figure 3 plots the time-series of the extracted variance swap rates on the return factors and illustrates the outcomes in each step of the extraction. The left panels present the case of constant asset factor exposures, the right panels the case of time-varying factor exposures. Appendix F describes the procedure in detail.[16]

[Figure 3 about here]

Table 4 reports the correlations between different variance swap rate components. For both indices, the correlation between the factor component in the index variance swap rate, $\beta'_{I,p,t}VS_{t,\tau}\beta_{I,p,t}$, and the index variance swap rate $VS_{I,p,t,\tau}$ is near perfect, reflecting the small role played by idiosyncratic risk in index variance swap rates. The correlation between the factor component in the index variance swap rates and the average variance swap rate of the index constituents, $\sum_n w_{n,p,t}VS_{n,t,\tau}$, is also high, with values of 88.62% for the S&P 100 index and 96.52% for the Nasdaq 100 index in the case with constant factor exposures, and 88.33% and 96.50% in the case with time-varying factor exposures.

[Table 4 about here]

Notably, a sizeable correlation is present between the factor component of the index variance swap rates and the average *idiosyncratic* variance swap rate of the index constituents, $\sum_n w_{n,p,t}VS_{n,\epsilon,t,\tau}$.[17] The correlation, which equals 52.99% for the S&P 100 index and 74.23% for the Nasdaq 100 index with constant factor exposures and 19.67% and 68.57% in the case with time-varying factor exposures, suggests that there exist common factors in assets' idiosyncratic variance swap rates, and

[16] A known issue in standard factor analysis is that estimated factor loadings and scores are unique only up to scale and rotation. As we show in Appendix F, when the factors have time-varying variances, the appropriate rotation can be identified. Specifically, the optimal rotation is the one that minimizes the time series standard deviation of local measures of the factor covariances. After rotating the factors in this fashion, we apply the methodology of Section 2.2 to extract the factor variance swap rates. Given the need to use two return factors, a limitation in our options data is that only two of the available stock indices exhibit sufficient heterogeneity in factor exposures (see footnote 11). Hence, we are able to identify factor variance swap rates on the two factors but not the covariance swap rate between them. Since the factor analysis of returns produces factors that are orthogonal and our rotation minimizes the time series variation of the factor covariance, we expect the covariance swap rate to be small throughout the sample and omitting it not to significantly affect our estimates of the factor variance swap rates. We therefore drop the factor covariance swap rate from our estimation problem.

[17] We use $VS_{n,\epsilon,t,\tau} = VS_{n,t,\tau} - \beta'_{n,t}VS_{t,\tau}\beta_{n,t}$ in the case with constant factor exposures and $VS_{n,\epsilon,t,\tau} = VS_{n,t,\tau} - \tilde{\beta}'_{n,t}VS_{t,\tau}\tilde{\beta}_{n,t} - \sum_j \mathrm{var}_t(\beta_{n,t+\tau/2}(j))VS^{jj}_{t,\tau}$ in the case with time-varying factor exposures. See Appendix A for the derivation of this expression.

that at least one of these factors is correlated with the factor variance swap rates. Further analysis confirms this intuition. While Section 3.3 has revealed common variation in total variance swap rates, a factor analysis of the panel of idiosyncratic variance swap rates yields that four factors account for 65.28% of the variation in idiosyncratic variance swap rates; these common idiosyncratic variance factors are strongly related to the variance swap rates on the return factors. The canonical correlation coefficients between the two sets of factors are 92.97% and 83.13%, revealing that two of the common idiosyncratic variance factors track the variance swap rates on the return factors.[18]

Does the portion of the common idiosyncratic variance factors unrelated to the return factor variance swap rates matter? To determine whether this is the case, we compare the average R^2 values of regressions of assets' idiosyncratic variance swap rates on the two return factor variance swap rates with those obtained by regressing them on the four common idiosyncratic variance factors. The average R^2 is 24.41% in the former case and 42.79% in the latter. When using time-varying factor exposures, the corresponding values are 22.82% and 44.66%. Thus, although the return factor variance swap rates have a strong impact on assets' idiosyncratic variance swap rates, common factors unrelated to the return factor variance swap rates play an important role in the cross-section of idiosyncratic variance swap rates.

4.2 Decomposition of variance risk premia

We are now ready to decompose the variance risk premia on individual names into their factor variance and idiosyncratic return variance components. From condition (9), total (instantaneous) variance risk premia equal the sum of systematic and idiosyncratic variance risk premia. As a result, one can split the variance swap return $r_{n,t,\tau}$ given by (20) as follows:

$$r_{n,t,\tau} = \beta'_{n,t} \underbrace{\frac{RV_{t,\tau} - VS_{t,\tau}}{VS_{n,t,\tau}}}_{\text{Factor variance swap return}} \beta_{n,t} + \underbrace{\frac{RV_{n,\epsilon,t,\tau} - VS_{n,\epsilon,t,\tau}}{VS_{n,t,\tau}}}_{\text{Idiosyncratic variance swap return}}, \tag{21}$$

where $RV_{t,\tau}$ denotes realized factor (co)variances and $RV_{n,\epsilon,t,\tau}$ individual assets' realized idiosyncratic variances.

[18] When considering the idiosyncratic variance swap rates computed using time-varying factor exposures, four factors explain 62.34% of the variation and the canonical correlation coefficients are 92.87% and 71.20%.

Table 5 reports monthly index-weighted average variance risk premia (VRP) for the index constituents of the S&P 100 and, respectively, Nasdaq 100 and the decomposition of total variance risk premia into systematic and idiosyncratic variance components for the entire sample period as well as split by calendar year.[19] *Specification 1* (on the left) assumes constant factor exposures β_n and *Specification 2* (on the right) allows for time-varying factor exposures $\beta_{n,t}$. For the constituents of either stock index, both the systematic and idiosyncratic components of the total variance risk premium are economically sizeable and statistically significant, but of opposite signs. For S&P stocks, the average monthly systematic variance risk premium is -19.73% (NW t-stat = -8.19) and the average idiosyncratic VRP is 23.43% (NW t-stat = 14.28). For Nasdaq stocks, the average systematic VRP is -11.99% (NW t-stat = -6.00) and the average idiosyncratic VRP 21.64% (NW t-stat = 12.99). Thus, the total variance risk premium is about zero for S&P 100 stocks because idiosyncratic and systematic variance risk premia roughly offset each other. By contrast, the idiosyncratic variance risk premium for Nasdaq constituents is about twice as large in absolute value as the average systematic component, resulting in a positive risk premium on total stock variance risk. Hence, by splitting variances and variance swap rates into systematic and idiosyncratic components, we uncover a negative risk premium on systematic variance and a *positive risk premium on idiosyncratic variance.*

[**Table 5 about here**]

These results are robust when we split the data by year. For the members of either index, there is sizeable variation in the variance risk premia across years, both for total variance and for the components. Nonetheless, a clear pattern emerges from Table 5. The systematic variance component is negative with two exceptions (years 2000 and 2008) and the idiosyncratic variance component is positive in every year, while the total variance risk premium is positive in about half of the years for both indices (seven out of fourteen years).[20] The results are also robust to modeling time-varying factor exposures, as can be seen in *Specification 2* reported in the second set of columns in Table 5.[21]

[19]The values reported for the entire sample differ slightly from the average of the yearly values because the data for 2009 ends on October 31.

[20]Even though average systematic and idiosyncratic variance risk premia have opposite signs, the correlation between both series is positive, with values of 45.07% for S&P 100 stocks and 33.01% for Nasdaq 100 stocks when using constant factor exposures, and of 20.69% and 32.41% when allowing for time-varying exposures.

[21]In the remainder of the paper, we present the results for the case allowing for time-varying factor exposures. The

These results suggest that index options are expensive because they hedge increases in factor variances and that *single-stock options are cheap once their exposure to the common return factors is accounted for.* That is, idiosyncratic variance swaps sell for less than their expected discounted payoffs.[22] The expected monthly return is around 20%, which suggests that an insurer against increases in idiosyncratic volatility loses substantial amounts on average.

4.3 Is common idiosyncratic variance risk a priced factor in the cross-section?

In the previous section we have found that long positions in idiosyncratic variance swaps earn substantial returns. One justification for these return patterns is a risk-based explanation. In Section 3.3, we have documented that there are common movements in the variances of idiosyncratic stock returns. We now explore whether common idiosyncratic variance risk ($CIVR$) is a priced factor in options. For this purpose, we construct proxies for common idiosyncratic variance risk factors ($CIVR$ factors) as the cross-sectional average idiosyncratic variance swap return on the index constituents for each of the indices. We then apply the two-stage Fama-MacBeth procedure to estimate factor loadings and risk premia. In addition to the $CIVR$ factors, we include the Fama-French and momentum factors ($FF4$) and, as suggested by Ang et al. (2006) and Carr and Wu (2009), proxies for systematic variance risk (two SVR factors measured by S&P and, respectively, Nasdaq index variance swap returns) in the following specification for expected excess returns:

$$E(r_{n,t,\tau} - r_{f,t}) = \sum_{i=1}^{4} \beta^i_{FF4}\lambda^i_{FF4} + \sum_{i=1}^{2} \beta^i_{SVR}\lambda^i_{SVR} + \sum_{i=1}^{2} \beta^i_{CIVR}\lambda^i_{CIVR}. \tag{22}$$

Table 6 reports the estimated risk premia (t-statistics are in parentheses). Consistent with the prior literature, we find that systematic variance risk is a priced factor in the cross-section and

results obtained when assuming constant factor exposures are similar.

[22] For a better understanding of option returns, it is instructive to contrast our results with those reported in the existing literature. As mentioned in the introduction, Carr and Wu (2009) find that only systematic variance carries a negative risk premium, while Driessen et al. (2009) find that variance does not carry a risk premium but correlation risk does. By accounting for the presence of common factors in asset returns, we show that both systematic and idiosyncratic variance are priced, but their risk premia have different signs. In terms of option prices, the Carr and Wu (2009) results say that index options are expensive (i.e., their implied variance exceeds the index's physical variance) because they allow hedging increases in market variance, and single-stock options are expensive to the extent (and only to the extent) that they are exposed to shifts in market variance. The Driessen et al. (2009) results mean that single-stock options are not expensive (i.e., their implied variances reflect the physical variance of the underlying stock), but index options are expensive because the index is subject to correlation shocks.

commands a negative risk premium, reflecting its nature as a hedge against economic downturns and crises. Both $CIVR$ factors are also significantly priced in the cross-section of equity option returns, with large positive prices of risk. In the next section, we investigate whether the asset pricing model (22) is able to capture the sort portfolio return patterns documented in Table 2.

[**Table 6 about here**]

4.4 Can priced idiosyncratic variance risk explain the cross-section?

We documented in Table 2 a number of regularities in the cross-section of equity option/variance swap returns. In particular, sort portfolios based on variance measures (ratio of historical variance to variance swap rate, historical variance, variance swap rate), past returns (variance swap return), and those based on stocks' systematic risk (exposure to S&P 100 and Nasdaq 100 index returns) earn substantial returns and in part have low turnover, suggesting a risk-based explanation based on the factor model (22).

Table 7 reports monthly abnormal portfolio returns (Alpha) and factor loadings for two expected return models, the Fama-French four-factor model ($FF4$) and the Fama-French model augmented by variance risk factors as in (22) ($FF4$+VR). The estimates are from time-series regressions for the $5-1$ long-short sort portfolio returns constructed in Table 2. The variance risk factors include the two SVR factors that proxy for systematic variance risk (measured by S&P and, respectively, Nasdaq index variance swap returns) and the two $CIVR$ factors that proxy for common idiosyncratic variance risk (measured as the cross-sectional average idiosyncratic variance swap return on the index constituents for each of the indices).

[**Table 7 about here**]

In Table 7, all sort portfolios exhibit large and significant abnormal returns when measured against the $FF4$ model, ranging from 8.70% to 18.30% per month. The estimates from the $FF4$+VR model, by contrast, reveal that the returns can almost entirely be attributed to the portfolios' exposures to the variance factors. Abnormal returns decline to between -2.56% and 1.94% per month and become insignificant in the $FF4$+VR model. These results show that options are exposed to risk factors that are not spanned by the Fama-French and momentum factors. Consistent with this interpretation, the factor loadings reported in Table 7 reveal that the sort portfolios load strongly on the variance risk factors.[23]

In summary, the risk factor model (22) can account for the cross-sectional pricing patterns documented in Table 2.

5 Why is Idiosyncratic Variance Risk Priced?

In this section, we explore a number of potential explanations for why the risk premium on systematic variance is negative and, in particular, that on idiosyncratic variance is positive. We consider the following candidate explanations: macroeconomic rationales (Merton's (1973) Intertemporal CAPM), market frictions (option illiquidity and hedging costs), and theories of financial intermediation under capital constraints (equilibrium price implications of negative skewness preferences, in particular hedging of corporate option compensation and nickel-picking investment strategies by investment funds).

5.1 Systematic and idiosyncratic variance as ICAPM state variables

A natural explanation for the results in Section 4.2 is that systematic and idiosyncratic variances are state variables whose innovations contain information about the future state of the economy and/or changes in investment opportunities. Campbell (1993) shows that conditioning variables that forecast the return or variance on the market portfolio will be priced. Conditioning variables for which positive shocks are associated with good (bad) news about future investment opportunities have a positive (negative) risk price so long as the coefficient of relative risk aversion exceeds unity. Thus, the ICAPM suggests that increases in systematic variance are bad news and increases in idiosyncratic variance are good news.[24] The remaining question is whether we can find direct

[23]When only the $FF4$ and systematic variance risk factors are included in the benchmark, abnormal returns on all sort portfolios are similar to the case where only the $FF4$ factors are included, and they remain highly significant. Thus, the $CIVR$ factors are the key driver of the performance of the sort portfolios.

[24]Campbell et al. (2001) provide evidence that idiosyncratic variances are countercyclical, positively related to market variance, and negatively predict GDP growth, suggesting that idiosyncratic variance should command a negative price of risk in equilibrium.

support for the predictive power of systematic and idiosyncratic variances?

Table 8 investigates the statistical relationship between market returns, market variance, idiosyncratic variance, and a number of macroeconomic variables. The dependent variables used in the various specifications across the columns are the quarterly market excess return $r_{M,t} - r_{f,t}$, market variance $\sigma^2_{M,t}$ (used as proxy for systematic variance), total variance (computed as the cross-sectional average of stocks' total variance), common idiosyncratic variance $\overline{\sigma}^2_{\epsilon,t}$ (computed as the cross-sectional average of idiosyncratic variance), quarterly GDP growth, investment growth, consumption growth, the 3-month T-bill rate, the term spread (computed as the difference between the yield on 10-year Treasury bonds and the 3-month T-bill rate), and the default spread (computed as the difference between the yield on BAA and AAA corporate bonds). Gross domestic product, investment, and consumption data are taken from the Federal Reserve Bank of St. Louis' FRED system, and interest rate data is from the Federal Reserve Statistical Release. We conduct the regressions using both levels (Panels A and C) and innovations (Panels B and D) of the dependent and independent variables. For each series, innovations are computed as the residuals from an AR specification with the number of lags selected optimally using Schwarz' Bayesian Information Criterion (BIC). The number of lags used when computing the innovations in each series is reported in the row labeled "AR Lags (BIC)" in Panels B and D. Panels A and B report estimates from contemporaneous regressions, and Panels C and D the results of predictive regressions for a one-quarter horizon.

[Table 8 about here]

The contemporaneous regressions reported in Panel A (on levels) and B (on innovations) confirm the strong leverage effect (negative contemporaneous relation between market return and market variance in columns 2-3) and the positive comovement in market and idiosyncratic variances documented in prior studies (columns 3 and 5). There is no statistical link between market returns and idiosyncratic variance after controlling for the correlation between market return and variance (columns 2 and 5). Now consider the macroeconomic variables. Market variance is negatively related to GDP growth (column 6) and to investment growth (column 7). There is also a negative relationship between market variance and consumption growth and the T-bill rate (columns 8 and

9), and a positive relationship with the term spread and the default spread (columns 10 and 11). The reverse contemporaneous relation holds between idiosyncratic variance and the macroeconomic indicators. In particular, idiosyncratic variance is positively correlated with GDP growth (t-stat = 1.35), investment growth (t-stat = 0.58), consumption growth (t-stat = 4.62), T-bill (t-stat = 3.46), and negatively correlated with term spread (t-stat = -1.13) and default spread (t-stat = -3.87). Table 8, Panel B reveals a similar picture for innovations with a few exceptions.[25]

The predictive regressions over a one-quarter horizon reported in Panels C and D reveal a different pattern.[26] Market variance does not significantly predict market returns, but predicts itself and negatively forecasts idiosyncratic variance, GDP growth, and investment growth. We find little forecasting power for future consumption or interest rate variables. By contrast, idiosyncratic variance is also positively autocorrelated, but does not significantly predict any of the macroeconomic indicators or interest rate variables. When considering innovations (Panel D), market variance negatively predicts GDP and investment growth. Idiosyncratic variance, again, does not have any predictive power. Summarizing, the results in Table 8 show that increases in market variance are indeed bad news, but we find no evidence that idiosyncratic variance predicts the future state of the economy.

For robustness, we also estimate a vector autoregression as suggested in Campbell (1993). We include in the system the monthly market excess return, $r_{M,t} - r_{f,t}$, market variance $\sigma^2_{M,t}$ (as proxy for systematic variance), and common idiosyncratic variance $\overline{\sigma}^2_{\epsilon,t}$ (computed as the cross-sectional average of idiosyncratic variance). Consistent with Table 8, the coefficients reported in Table 9 indicate that neither systematic variance nor idiosyncratic variance are related to future market returns. Market variance is positively related to future market variance, but idiosyncratic variance is not. Thus, increases in systematic variance are bad news about future investment opportunities, while increases in idiosyncratic variance seem irrelevant.

[Table 9 about here]

[25]The estimates in Panel B differ from Panel A in that the relationships between market variance and GDP growth, investment growth and, respectively, the term spread become statistically insignificant. In addition, there is a negative relationship between idiosyncratic variance innovations and the term spread.

[26]In Panel C, in order to account for the autocorrelation of the dependent variable, we include as regressor the lagged dependent variable up to a number of lags selected optimally using Schwarz' Bayesian Information Criterion (BIC). The number of lags used is reported in the row labeled "AR Lags (BIC)" in Panel C.

In summary, we find evidence of contemporaneous correlation between systematic and idiosyncratic variances and several macroeconomic indicators, and the sign of the correlations differ between systematic and idiosyncratic variances. However, we find no significant predictive power of idiosyncratic variances for indicators of macroeconomic conditions. Thus, the results in Tables 8 and 9 confirm that intertemporal hedging demands by investors can account for the negative risk premium on systematic variance, but it seems unlikely that they generate the observed positive price of idiosyncratic variance risk.

5.2 The effect of option illiquidity and hedging costs on variance risk premia

An alternative is that the positive idiosyncratic variance risk premium reflect compensation for the illiquidity of individual stock options or the difficulty in delta-hedging them. Market makers have been shown to be net long individual stock options (Garleanu, Pedersen, and Poteshman (2009)) and may therefore be willing to pay less for options that are more difficult to (delta-)hedge. In this case, we would expect a positive relationship between the variance risk premium and measures of market imperfections.

In Table 10, we conduct Fama-MacBeth regressions of variance risk premia on various characteristics and proxies for market frictions to investigate if frictional costs explain our findings. We include the quoted bid-ask spread on the underlying (in percent), the quoted bid-ask spread on options (average of the bid-ask spreads on at-the-money or, alternatively, out-of-the-money put and call options in percent), trading volume in the underlying and the options, and option open interest as explanatory variables in our regression specification. Additional control variables are also included in the three specifications.

[Table 10 about here]

The results reveal that variance risk premia are largely unrelated to bid-ask spreads on the underlying stock and on options. This result holds both for the total variance risk premium (left columns) and the idiosyncratic variance risk premium (right columns). Stock volume is positively related to the variance risk premium, while option volume has a hump-shaped impact. In summary, there is little evidence that the positive sign of the idiosyncratic variance risk premium represents compensation for the illiquidity of stock options or the costs involved in hedging them.

5.3 Equilibrium price implications of financial intermediation under capital constraints

Last, we explore the importance of capital-constrained financial intermediaries in determining risk compensation in the options market. Financial intermediaries play a pivotal role as counterparties in the options market. They provide liquidity to hedgers and speculators and absorb much of the demand and supply from investors. Idiosyncratic movements in the variances of idiosyncratic returns are diversified away in a dealer's large portfolio of options. What remains is the risk that the variances of idiosyncratic returns move in a systematic way. Intermediaries, hence, cannot hedge options perfectly and, as a result, are sensitive to risk. Appendix G develops a simple model of option market-making that formalizes this intuition and allows to rationalize the estimated variance risk premia. Common idiosyncratic variance risk commands a positive risk premium in equilibrium to the extent that investors are net suppliers of individual stock options.[27]

Many investor groups have indirect preferences for negative skewness and, therefore, supply individual stock options—generating supply pressure in single-stock options.[28] For instance, a prominent hedge fund strategy is to short individual stock variance, generating a high propensity of small gains and infrequent large losses (known as "picking up nickels in front of a steamroller").[29]

[27] Using a unique dataset, Garleanu, Pedersen, and Poteshman (2009) provide direct evidence that end-users of options on individual names are net short while option market makers are net long. Further, they show in cross-sectional tests that end-users' net demand impacts the expensiveness of single-stock options.

[28] Scott and Horvath (1980) demonstrate that preferences for moments of higher order than the variance yield "lotto" behavior, i.e., investors prefer positive skewness in return distributions. Mitton and Vorkink (2007) show that heterogeneity in preferences is required so that skewness induces investors to underdiversify in equilibrium.

[29] Malliaris and Yan (2010) show that reputation concerns induce fund managers to adopt strategies with negatively skewed payoffs, even if such strategies generate inferior returns. Such "nickel-picking" strategies make money most of the time and suffer infrequent large losses. To understand why, consider a mutual fund that engages in covered call writing, a strategy that is explicitly allowed under SEC regulations—as described in CBOE (2001), SEC regulations and no-action letters provide that a mutual fund seeking to take a short option position must either (1) hold the underlying security or an offsetting option position, i.e., "cover" the option position, or (2) set aside in a segregated, custodial account consisting of cash, U.S. government securities, or high-grade debt securities in an amount at least equal in value to the optioned securities, i.e., "segregation of assets". Such a fund would outperform funds that do not most of the time, except in periods where stocks do especially well. To the extent that this return pattern has a positive impact on fund flows (which will occur if investors value the extra return in most years more than the return they give up in good stock market years, or if they do not realize that the outperformance in most years comes at a cost), the strategy will be attractive to the fund manager. Malliaris and Yan (2010) show that four out of the ten style indices in the Credit Suisse/Tremont Hedge Fund Index, representing more than 40% of total hedge fund assets, have negatively skewed returns at the 5% level.

Covered call writing is also widespread among individual investors. Using accounts from a sample of retail investors at a discount brokerage house, Lakonishok et al. (2007) document that a large fraction of call writing is part of covered-call strategies. Principal-agent relations in corporations are another source. Corporate managers and employees receive substantial option-based compensation. They have, thus, an incentive to offset the convexity in their payoffs by shorting exchange-traded options on their employer's stock. While such trades are often prohibited by employment contracts, it is questionable to what extent such provisions are enforceable.[30] A natural question is whether option issues by firms are sizeable enough to impact on prices of exchange-traded options? Between 1996 and 2009, the outstanding amount of company-issued options constitutes on average about 77% (139%) of the call (put) open interest in front-month exchange-traded options closest to at-the-money, and about 25% (58%) of the total call (put) open interest of all front-month exchange-traded options.[31] All of these considerations suggest that there exists a large supply of single-stock options by investors that financial intermediaries have to absorb.

The equilibrium pricing conditions of the model derived in Appendix G yield additional testable restrictions. In particular, (1) the cross-section of variance risk premia reflects the asset's exposure to the common idiosyncratic variance factor(s), the price of risk for common idiosyncratic variance is larger at times (2) when the total net supply of stock options is larger and (3) when the riskiness of common idiosyncratic variance is larger, and an asset's variance risk premium is higher (4) the larger the net supply of variance for that asset and (5) the more variable the asset's idiosyncratic variance.

Testing the cross-sectional implications: In our empirical study, we capture several sources of supply by measuring option writing by investment funds to enhance yields and attract fund flows and by holders of firm-issued options such as employee stock options. Our proxies for option

[30] A number of financial firms offer covered call writing and escrow services to corporate managers. The Securities Exchange Act of 1934 and SEC regulations do not prohibit managers from hedging their employee stock and stock options, so long as the delta of their overall position—including employee stock, employee stock options, exchange-traded stock, and exchange-traded options—remains positive. For details, see the Securities and Exchange Commission in its opinion letter "Response of the Office of Chief Counsel, Division of Corporation Finance, Re: Credit Suisse First Boston ("CSFB") Incoming letter dated March 16, 2004."

[31] To compute these numbers, we compare for each firm in our sample the number of company-issued options outstanding at the end of each year (Compustat variable *optosey*) to the open interest on exchange-traded options reported in OptionMetrics. The magnitudes we obtain appear sufficient to generate a first-order price impact.

supply are, first, the ratio of the number of shares held by mutual funds at the end of each quarter (obtained from the ThomsonReuters mutual fund holdings database) divided by the number of shares outstanding. Second, we use the ratio of the number of firm-issued options outstanding at the end of each year (Compustat variable *optosey*) divided by the number of shares outstanding.

We estimate the equilibrium pricing condition (G.14) using Fama-MacBeth regressions of the individual stocks' realized idiosyncratic variance risk premia on the following explanatory variables: exposure to the market, value, size, and momentum factors; exposure to the common idiosyncratic variance risk factor (constructed as the cross-sectional average of individual stocks' realized idiosyncratic variance risk premium); our proxies for supply pressure (plus indicator variables for when the variables cannot be constructed); and the riskiness of each stock's "truly idiosyncratic" variance (computed as the time-series variance of the residuals from the first-pass regression of each stock's variance risk premium on the four Fama-French factors plus the common idiosyncratic variance factor).

Table 11 reports the estimation results. The left (right) two columns investigate total (idiosyncratic) variance risk premia. Consistent with our predictions, idiosyncratic variance risk premia are larger for stocks that have larger exposure to common idiosyncratic variance, larger option compensation, larger mutual fund holdings, and more variable "truly idiosyncratic" variance. All coefficients are statistically significant.[32]

[Table 11 about here]

Testing the time-series implications: The time-series predictions of the model are that the risk premium on common idiosyncratic variance should be larger in periods when the aggregate net supply of single-stock options is larger and when the riskiness of common idiosyncratic variance is larger.[33] To test if there a positive relationship between the riskiness of common idiosyncratic

[32] To check for robustness of our results, the left two columns in Table 11 report the estimation results for the total variance risk premium on the same set of explanatory variables (this specification is exact if market makers cannot hedge their variance exposure using index options), except that we replace the common idiosyncratic variance factor with a common variance factor—constructed as the cross-sectional average of stocks' total variance risk premium. As before, variance risk premia are positively related to stocks' exposure to common variance and to option compensation. The coefficients on mutual fund holdings and "truly idiosyncratic" variance risk are, however, insignificant.

[33] Data on the number of firm-issued options outstanding and on mutual fund holdings are only available on a yearly and quarterly basis, respectively, and the aggregate series are highly persistent. This persistence, together with entry into option market making over the medium run, makes the relationship between aggregate supply and

variances and the risk premium on common idiosyncratic variance, we estimate a GARCH-in-mean model on the cross-sectional average idiosyncratic variance risk premium. The estimation results are presented in Table 12. As predicted by our model, the in-mean effect in the variance risk premium is positive and highly significant. This holds true for both the total variance risk premium (left) and the idiosyncratic variance risk premium (right).

[Table 12 about here]

In summary, the estimates reveal that the positive sign of the idiosyncratic variance risk premium is consistent with supply pressure in single-stock options from investors (caused by option compensation and, to a lesser extent, by covered call writing by mutual funds). Market makers need to be compensated for absorbing the supply. Consistent with this hypothesis, we find that idiosyncratic variance risk premia are higher, the greater the number of firm-issued options outstanding and the larger the mutual fund ownership. The idiosyncratic variance risk premium is more strongly positive, the larger the supply of such options, the larger the exposure of the underlying stock to shifts in common idiosyncratic variance, and the greater the riskiness of a stock's "truly idiosyncratic" variance.

6 Robustness

We have conducted a number of robustness checks. This section discusses the robustness of our findings to the modeling assumptions and the limitations of the data.

6.1 Modeling assumptions and implementation frictions

Constant versus time-varying factor exposures: We have performed the analysis in Sections 4.3, 4.4, and 5 assuming either constant factor exposures or time-varying factor exposures. We found very similar results in both cases and, therefore, report the estimates assuming time-varying factor exposures.

variance risk premia predicted by the model difficult to document. We have tried various specifications and were not able to document a robust impact of our measures of aggregate supply on the average idiosyncratic variance risk premium.

Transaction costs: Transaction costs may limit market participants' ability to arbitrage any variance swap/option mispricing and to take exposures to different components of variance risk. The bid and ask quotes available in OptionMetrics are only indicative end-of-day quotes, hence of limited informativeness about actual transaction costs. While we have no information on spreads on OTC single-stock variance swap rates, bid-ask spreads on OTC index variance swap rate quotes from major broker dealers are on average around 50-100 basis points (Egloff, Leippold, and Wu (2009)) and, hence, an order of magnitude lower than the returns documented in this paper.

Discrete option strike prices and jumps in underlying prices: The replication of variance swaps using a portfolio of out-of-the-money options and delta-hedging in the underlying is exact only if options with an unlimited number of strikes are available and underlying asset prices do not jump. Nonetheless, Jiang and Tian (2005) show that variance swap rates can be computed accurately from option prices even if the underlying price process jumps and a limited number of strikes prices are available. Carr and Wu (2009) show that the approximation error introduced by jumps is of third order, and Broadie and Jain (2007) show that under realistic parameterizations the jump-induced error in variance swap rates computed using the replicating portfolio is less than 2%. Dividends or stochastic interest rates are another source of approximation error. Though, Torné (2009) shows that the absolute error from these issues is less than 1%.

As a final check, we have used a model-free approach to quantify variance risk premia. The next section discusses the results in more detail.

6.2 Model-free variance risk premia

A potential concern is whether our findings about variance and correlation risk premia are driven by the factor model assumptions or the empirical implementation. In the following, we provide support that our findings hold using a model-free approach, by measuring the profitability of different dispersion trading strategies. Such strategies involve taking long positions in options (or variance swaps) on the index constituents and a short position in options (or variance swaps) on the underlying index.[34]

A simple experiment shows that the view that correlation risk is priced but variance risk is not cannot account for the profitability of dispersion trades. Consider the returns of a dispersion trading strategy in which variance swaps on individual stocks are purchased in proportion to the stocks' weights in the index and in which the size of the short position in the index variance swap is set such that the portfolio has zero exposure to the returns on the index variance swap. Such a strategy can easily be constructed as described in Appendix H. Such a portfolio should earn negligible returns on average if the source of the profitability of dispersion trades is the correlation risk premium.

Table 13, Panel A, reports summary statistics for the return characteristics of dispersion trading strategies constructed in this way, separately for the S&P 100 and, respectively, the Nasdaq 100 index. In spite of zero exposure to index variance swap returns, the strategy is highly profitable, generating Sharpe ratios of 1.7 for S&P and 2.2 for Nasdaq. Notably, as can be seen in Panel B of the table, strategies constructed to be uncorrelated with the average constituent variance swap return are profitable as well. The corresponding Sharpe ratios are 1.8 for S&P and 2 for Nasdaq.

[Table 13 about here]

These results constitute a challenge not only for the view that variance risk is not priced and correlation risk is, but also for the view that only systematic (i.e., market) variance risk is priced—if the latter were the case, the strategies in Panel A should be unprofitable. The results in Table 13 can be reconciled once one recognizes that both factor and idiosyncratic variance risk are priced and dispersion trading strategies, rather than earning the correlation risk premium, earn a combination of the positive idiosyncratic variance risk premia on the index constituents and the negative variance risk premium on the common return factors. Appendix H provides a formal proof.

[34] The common view among practitioners is that such trades earn the correlation risk premium. The intuition is that the short side of the trade is exposed to both variance and correlation risk, while the long side is exposed to variance risk only. So long as variance risk is unpriced, the profits are due to correlation risk premia.

7 Conclusion

We document that common movements in the variances of idiosyncratic returns are strongly priced risk factors in the cross-section of equity option returns and not accounted for by the Fama-French four-factor model. For this purpose, we develop a financial market model in which systematic and idiosyncratic variance risk are allowed to be priced. Our tractable framework allows separately identifying variance swap rates and variance risk premia on the systematic and idiosyncratic components of asset returns. Correlation risk is a composite of return factor variance risk and idiosyncratic return variance risk in this setting.

In our empirical analysis using a large cross-section of equity options—spanning the S&P 100 and Nasdaq 100 indices, we find that the market price of risk for the variance of systematic returns is negative, while that for the variance of idiosyncratic returns is positive and sizeable. The differential pricing of systematic and idiosyncratic variance risk explains several phenomena, including (1) the relative expensiveness of index options and cheapness of individual options, (2) the sizeable cross-sectional variation in risk premia on individual stock variances, (3) the volatility mispricing puzzle documented by Goyal and Saretto (2009), and (4) the substantial returns earned on various option portfolio strategies that we document in the paper. We also show that dispersion trading strategies commonly used by hedge funds earn a combination of the negative risk premium on factor variance risk and the positive risk premium on idiosyncratic variance risk.

We find little evidence for ICAPM- and liquidity-based explanations of the observed patterns. In order to rationalize the estimated risk premia, we embed the financial market assumptions in a theory of financial intermediation under capital constraints. The model predictions find support in the data and can account for the observed positive market price of idiosyncratic variance risk. The results are robust to a number of additional tests.

Appendix

A Methodology for Factor and Idiosyncratic Variance Swap Extraction with Time-Varying Factor Exposures and Parameter Uncertainty

In this appendix, we demonstrate that the approach laid out in Section 2.2 can be applied with minor modifications to situations with time-varying factor exposures and parameter uncertainty. Sufficient conditions are that the following three assumptions hold (under Q):

1. Changes in factor exposures are uncorrelated with factor variances and covariances;
2. Changes in factor exposures are uncorrelated across assets and factors;
3. Assets' factor exposures follow a random walk without drift.

With time-varying factor exposures (and irrespective of whether the above assumptions are met), the variance swap rate on an individual asset and, respectively, the variance swap rate on index p (again assuming constant index weights over the contract life) are:

$$\begin{aligned} VS_{n,t,\tau} &= \frac{1}{\tau}E_t^Q[\int_t^{t+\tau} \sigma_{n,u}^2 du] = \frac{1}{\tau}E_t^Q[\int_t^{t+\tau} (\beta_{n,u}'\Sigma_u\beta_{n,u} + V_{n,\epsilon,u})du], \qquad \text{(A.1)} \\ VS_{I,p,t,\tau} &= \frac{1}{\tau}E_t^Q[\int_t^{t+\tau} \sigma_{I,p,u}^2 du] \\ &= \frac{1}{\tau}E_t^Q[\int_t^{t+\tau} ((\sum_{n=1}^N w_{n,p,t}\beta_{n,u}')\Sigma_u(\sum_{n=1}^N w_{n,p,t}\beta_{n,u}) + \sum_{n=1}^N w_{n,p,t}^2 V_{n,\epsilon,u})du]\ . \qquad \text{(A.2)} \end{aligned}$$

The *adjusted* index variance swap rate is

$$\begin{aligned} y_{p,t} &\equiv VS_{I,p,t,\tau} - \sum_{n=1}^N w_{n,p,t}^2 VS_{n,t,\tau} \\ &= \frac{1}{\tau}E_t^Q[\int_t^{t+\tau} ((\sum_{n=1}^N w_{n,p,t}\beta_{n,u}')\Sigma_u(\sum_{n=1}^N w_{n,p,t}\beta_{n,u}) - \sum_{n=1}^N w_{n,p,t}^2\beta_{n,u}'\Sigma_u\beta_{n,u})du]\ . \qquad \text{(A.3)} \end{aligned}$$

Dropping matrix notation for convenience and using i and j to index the factors yields (the second line follows from assumption 1):

$$\begin{aligned} y_{p,t} &= \frac{1}{\tau}E_t^Q[\int_t^{t+\tau} (\sum_j((\sum_{n=1}^N w_{n,p,t}\beta_{n,u}(j))^2 - \sum_{n=1}^N w_{n,p,t}^2\beta_{n,u}(j)^2)\Sigma_u^{jj})du] \qquad \text{(A.4)} \\ &+\frac{2}{\tau}E_t^Q[\int_t^{t+\tau} (\sum_i\sum_{j\neq i}((\sum_n w_{n,p,t}\beta_{n,u}(i))(\sum_{n=1}^N w_{n,p,t}\beta_{n,u}(j)) - \sum_{n=1}^N w_{n,p,t}^2\beta_{n,u}(i)\beta_{n,u}(j))\Sigma_u^{ij})du] \\ &= \frac{1}{\tau}\int_t^{t+\tau} (\sum_j E_t^Q[(\sum_{n=1}^N w_{n,p,t}\beta_{n,u}(j))^2 - \sum_{n=1}^N w_{n,p,t}^2\beta_{n,u}(j)^2]E_t^Q[\Sigma_u^{jj}])du \qquad \text{(A.5)} \\ &+\frac{2}{\tau}\int_t^{t+\tau} (\sum_i\sum_{j\neq i} E_t^Q[(\sum_{n=1}^N w_{n,p,t}\beta_{n,u}(i))(\sum_{n=1}^N w_{n,p,t}\beta_{n,u}(j)) - \sum_{n=1}^N w_{n,p,t}^2\beta_{n,u}(i)\beta_{n,u}(j)]E_t^Q[\Sigma_u^{ij}])du \end{aligned}$$

Using assumption 2, we have

$$\begin{aligned} E_t^Q[(\sum_{n=1}^N w_{n,p,t}\beta_{n,u}(j))^2 - \sum_{n=1}^N w_{n,p,t}^2\beta_{n,u}(j)^2] &= (E_t^Q[\sum_{n=1}^N w_{n,p,t}\beta_{n,u}(j)])^2 + \sum_{n=1}^N w_{n,p,t}^2 \text{var}_t^Q(\beta_{n,u}(j)) \\ &\quad - \sum_{n=1}^N w_{n,p,t}^2((E_t^Q[\beta_{n,u}(j)])^2 + \text{var}_t^Q(\beta_{n,u}(j))) \qquad (A.6) \\ &= (E_t^Q[\sum_{n=1}^N w_{n,p,t}\beta_{n,u}(j)])^2 - \sum_{n=1}^N w_{n,p,t}^2(E_t^Q[\beta_{n,u}(j)])^2 \end{aligned}$$

and

$$\begin{aligned} &E_t^Q[(\sum_{n=1}^N w_{n,p,t}\beta_{n,u}(i))(\sum_{n=1}^N w_{n,p,t}\beta_{n,u}(j)) - \sum_{n=1}^N w_{n,p,t}^2\beta_{n,u}(i)\beta_{n,u}(j)] \\ &\quad = E_t^Q[\sum_{n=1}^N w_{n,p,t}\beta_{n,u}(i)]E_t^Q[\sum_{n} w_{n,p,t}\beta_{n,u}(j)] - \sum_{n=1}^N w_{n,p,t}^2 E_t^Q[\beta_{n,u}(i)]E_t^Q[\beta_{n,u}(j)] \qquad (A.7) \end{aligned}$$

Finally, we use assumption 3 to replace the expectations of future factor exposures with their current estimates (denoted using tildes), yielding

$$E_t^Q[(\sum_{n=1}^N w_{n,p,t}\beta_{n,u}(j))^2 - \sum_{n=1}^N w_{n,p,t}^2\beta_{n,u}(j)^2] = (\sum_{n=1}^N w_{n,p,t}\tilde{\beta}_{n,t}(j))^2 - \sum_{n=1}^N w_{n,p,t}^2\tilde{\beta}_{n,t}(j)^2 \qquad (A.8)$$

and

$$\begin{aligned} &E_t^Q[(\sum_{n=1}^N w_{n,p,t}\beta_{n,u}(i))(\sum_{n=1}^N w_{n,p,t}\beta_{n,u}(j)) - \sum_{n=1}^N w_{n,p,t}^2\beta_{n,u}(i)\beta_{n,u}(j)] \\ &\quad = (\sum_{n=1}^N w_{n,p,t}\tilde{\beta}_{n,t}(i))(\sum_{n=1}^N w_{n,p,t}\tilde{\beta}_{n,t}(j)) - \sum_{n=1}^N w_{n,p,t}^2\tilde{\beta}_{n,t}(i)\tilde{\beta}_{n,t}(j) \qquad (A.9) \end{aligned}$$

Inserting these expressions back into the expression for the adjusted index variance swap rate $y_{p,t}$ yields

$$\begin{aligned} y_{p,t} &= \frac{1}{\tau}E_t^Q[\int_t^{t+\tau}(\sum_j((\sum_{n=1}^N w_{n,p,t}\tilde{\beta}_{n,t}(j))^2 - \sum_{n=1}^N w_{n,p,t}^2\tilde{\beta}_{n,t}(j)^2)\Sigma_u^{jj})du] \\ &\quad + \frac{2}{\tau}E_t^Q[\int_t^{t+\tau}(\sum_i\sum_{j\neq i}((\sum_{n=1}^N w_{n,p,t}\tilde{\beta}_{n,t}(i))(\sum_{n=1}^N w_{n,p,t}\tilde{\beta}_{n,t}(j)) - \sum_{n=1}^N w_{n,p,t}^2\tilde{\beta}_{n,t}(i)\tilde{\beta}_{n,t}(j))\Sigma_u^{ij})du] \\ &= \tilde{\beta}'_{I,p,t}VS_{t,\tau}\tilde{\beta}_{I,p,t} - \sum_{n=1}^N \tilde{\beta}'_{n,t}VS_{t,\tau}\tilde{\beta}_{n,t} \qquad (A.10) \end{aligned}$$

Thus, when the factor exposures are time-varying and/or when there is uncertainty about their true value, (15) still holds; it suffices to compute $A_{p,j,t}$ and $B_{p,i,j,t}$ in (16) using estimated factor exposures $\tilde{\beta}_{n,t}$.

Once the factor variance swap rates have been computed, one can estimate the idiosyncratic variance swap rate from the expression

$$VS_{n,\epsilon,t,\tau} = VS_{n,t,\tau} - \frac{1}{\tau}E_t^Q[\int_t^{t+\tau}\beta'_{n,u}\Sigma_u\beta_{n,u}du]. \qquad (A.11)$$

Using the above assumptions, this expression can be rewritten as

$$
\begin{aligned}
VS_{n,\epsilon,t,\tau} &= VS_{n,t,\tau} - \frac{1}{\tau} E_t^Q [\int_t^{t+\tau} (\sum_j \beta_{n,u}(j)^2 \Sigma_u^{jj} + 2 \sum_i \sum_{j \neq i} \beta_{n,u}(i) \beta_{n,u}(j) \Sigma_u^{ij}) du] \\
&= VS_{n,t,\tau} - \frac{1}{\tau} \int_t^{t+\tau} (\sum_j (\tilde{\beta}_{n,t}(j)^2 + \mathrm{var}_t(\beta_{n,u}(j))) E_t^Q [\Sigma_u^{jj}] + 2 \sum_i \sum_{j \neq i} \tilde{\beta}_{n,t}(i) \tilde{\beta}_{n,t}(j) E_t^Q [\Sigma_u^{ij}]) du \\
&= VS_{n,t,\tau} - \tilde{\beta}'_{n,t} VS_{t,\tau} \tilde{\beta}_{n,t} - \sum_j \frac{1}{\tau} \int_t^{t+\tau} \mathrm{var}_t(\beta_{n,u}(j)) E_t^Q [\Sigma_u^{jj}] du. \qquad \text{(A.12)}
\end{aligned}
$$

By the mean value theorem, there exists a value of u in $[t, t+\tau]$, denoted $\overline{u}$, such that

$$
\int_t^{t+\tau} \mathrm{var}_t(\beta_{n,u}(j)) E_t^Q [\Sigma_u^{jj}] du = \mathrm{var}_t(\beta_{n,\overline{u}}(j)) \int_t^{t+\tau} E_t^Q [\Sigma_u^{jj}] du. \qquad \text{(A.13)}
$$

In our implementation, since $\mathrm{var}_t(\beta_{n,u}(j))$ is linear in u, we will approximate this expression using $\overline{u} = t+\tau/2$. Thus, we extract the idiosyncratic variance swap rate using

$$
VS_{n,\epsilon,t,\tau} = VS_{n,t,\tau} - \tilde{\beta}'_{n,t} VS_{t,\tau} \tilde{\beta}_{n,t} - \sum_j \mathrm{var}_t(\beta_{n,t+\tau/2}(j)) VS_{t,\tau}^{jj} \;. \qquad \text{(A.14)}
$$

B Approximation Accuracy in Expression (8)

In order to assess the accuracy of the approximation in expression (8), we compare the variance swap rates obtained from (8) with those obtained by simulating the system and thereby accounting for the random variation in weights. We do this in a parametric setting with a single common factor. The variance of the factor is assumed to follow a CIR process:

$$
dV_t = \kappa^Q (\overline{v}^Q - V_t) dt + \sigma \sqrt{V_t} dZ_{V,t}^Q \;. \qquad \text{(B.1)}
$$

The correlation between the factor and variance innovations is denoted by ρ.

We take the parameter values of the factor's variance process to be those estimated by Aït-Sahalia and Kimmel (2007) for the S&P 500 index using daily data for the period from January 2, 1990 until September 30, 2003, namely $\kappa = 5.07$, $\overline{v} = 0.0457$, $\sigma = 0.48$ and $\rho = -0.767$ (see Table 6, column (2) of their paper). We assume that the variances of the idiosyncratic noise terms follow independent square root processes with the same parameters as the factor variance. We assume that the index comprises 100 securities with identical initial weights and spread the exposure of the individual assets to the common factor uniformly around 1 using values from 0.505 to 1.495. This allows for heterogeneity in factor exposures, while guaranteeing an average exposure of 1. We consider a variance swap with a maturity of three months (the approximation is more accurate than reported below for shorter maturities), and compute the simulation-based variance swap rate using 20,000 simulation runs and a 1-day discretization interval.

The upper panel of Figure 4 shows the variance swap rates obtained from the simulation and using the approximation (8) for initial variances between 10% and 200% of the long-term mean estimated by Aït-Sahalia and Kimmel (2007) (we use the same initial variances for the factor and all assets' idiosyncratic noises). The lower panel reports the relative approximation errors. Observe that the approximation is almost indistinguishable from the variance swap rate obtained from the simulation. The approximation has a slight downward bias for very low initial variances, and a slight upward bias for very large initial variances. However, even in the worst cases, the approximation error is of the order of 1%. We conclude that the approximation (8) is quite accurate.

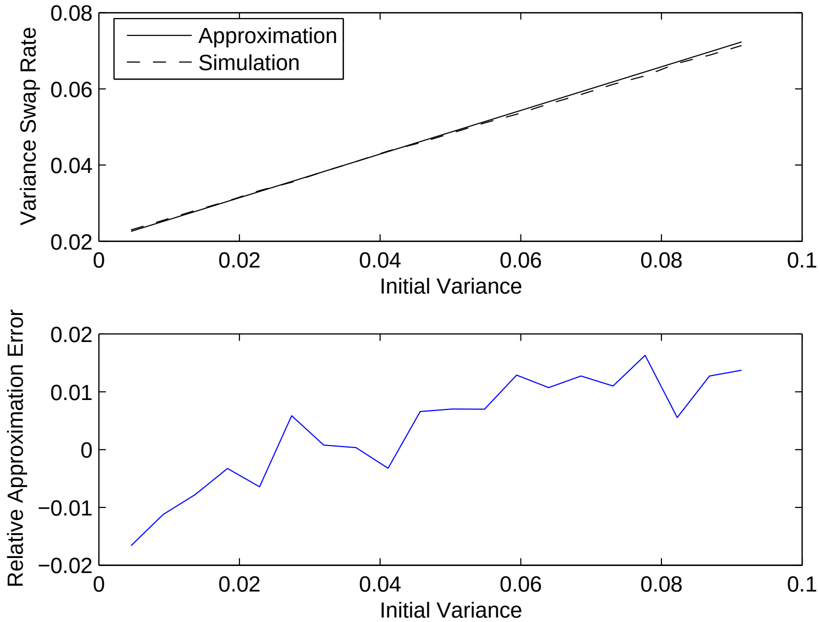

Figure 4: Accuracy of the index variance swap rate approximation (8).

The figure compares the index variance swap rates obtained from the approximation (8) with those obtained by simulating the system and thereby accounting for the random variation in index weights. The variances of the return factor and each stock's idiosyncratic return are assumed to follow CIR processes with the parameters estimated by Aït-Sahalia and Kimmel (2007).

[Figure 4 about here]

C Reconciling Variance and Correlation Risk Premia

It is possible to reconcile expression (10) with the expression for the index variance risk premium derived by Driessen et al. (2009). Letting $\rho_{m,n,t}$ denote the instantaneous return correlation between assets m and n, Driessen et al. (2009) show that the index variance risk premium is given by

$$VRP_{I,p,t} = \sum_{n=1}^{N} x_{n,p,t} VRP_{n,t} + \sum_{n=1}^{N} \sum_{\substack{m=1 \\ m \neq n}}^{N} w_{n,p,t} w_{m,p,t} \sigma_{n,t} \sigma_{m,t} (E_t^Q[d\rho_{m,n,t}] - E_t^P[d\rho_{m,n,t}]) \ , \quad \text{(C.1)}$$

where $x_{n,p,t} \equiv w_{n,p,t}^2 + \sum_{m \neq n} w_{n,p,t} w_{m,p,t} \rho_{m,n,t} \frac{\sigma_{m,t}}{\sigma_{n,t}}$. Thus, an alternative representation of (10) is that the variance risk premium on the index is composed of variance risk premia on individual assets, $VRP_{n,t}$, and correlation risk premia, $E_t^Q[d\rho_{m,n,t}] - E_t^P[d\rho_{m,n,t}]$. Expressions (10) and (C.1) can be shown to be equivalent once one accounts for the relationship between correlation risk premia and variance risk premia (11). Indeed, inserting (9, 11) into (C.1), using the fact that $\beta_{I,p,t} = \sum_{n=1}^{N} w_{n,p,t}\beta_{n,t}$ and simplifying yields (10):

$$\begin{aligned} VRP_{I,p,t} &= \sum_{n=1}^{N} w_{n,p,t}^2 VRP_{n,t} + \sum_{n=1}^{N} \sum_{\substack{m=1 \\ m\neq n}}^{N} w_{n,p,t} w_{m,p,t} \beta_{m,t}' VRP_t \beta_{n,t} \\ &= \beta_{I,p,t}' VRP_t \beta_{I,p,t} + \sum_{n=1}^{N} w_{n,p,t}^2 VRP_{n,\epsilon,t}. \end{aligned} \tag{C.2}$$

D Sample Selection, Index Constituents, and Index Weights

For both the S&P 100 index and the Nasdaq 100 index, we obtain historical index weights on each trading day in the sample period as follows. First, we compute the weight of each stock in each index on January 2, 1996, on each date in which constituent changes occurred, as well as on the regular quarterly index rebalance dates, which occur on the third Friday of March, June, September and December.[35] Starting from each of these rebalance or constituent change dates, we then compute the weights on the next trading day by multiplying them with one plus each stock's realized return and normalizing them such that they sum to unity. We do this until we reach the next rebalance or constituent change date.

For the S&P 100 index, we obtain the list of the constituents on January 2, 1996, the list of index constituent changes that took place during our sample period (there were 84 such changes), and the dates at which they occurred directly from Standard and Poor's. For the period from January 2, 1996 to December 31, 2000, historical weights for the S&P 100 index are not available at reasonable cost, so we compute the weights on each rebalance or constituent change date by normalizing the S&P 500 index weights, which are available from Bloomberg. This approach is accurate because Standard and Poor's accounts for free float, dual classes of stock, etc. in the same way for both indices. For the period from January 2, 2001 to September 30, 2008, we obtain the exact index weights on each rebalance or constituent change date directly from Bloomberg. For the period from October 1, 2008 to October 31, 2009, we compute the weights on each rebalance or constituent change date based on stocks' market capitalization.

For the Nasdaq 100 index, we obtain the list of constituent changes during our sample period (there were 227 such changes) and their dates from the Nasdaq website.[36] For the period from January 2, 1996 to December 31, 2000, exact index weights are not available at reasonable cost, so we estimate them based on the market capitalization of the constituent stocks. For the period from January 2, 2001 to October 31, 2009, we obtain the exact index weights directly from Bloomberg.

E Stock Return Factor Decomposition and Selection of the Number of Factors

In this appendix we ascertain whether a factor model with a small number of return factors is sufficiently accurate for our analysis. This is important in our case because option data are available on only a limited number of indices (see Footnote 11). For our return model to be well specified, three requirements need to be met: First, the latent return factors need to reproduce the time series of index returns; second, the variance of these factors needs to capture realized index variances and their movement through time; finally, the factors need to account for correlations in individual asset returns and their movement through time. We find that a factor model with two common return factors meets all three requirements.

[35] For the S&P 100 index, most index constituent changes do not occur on the regular quarterly rebalance dates, which deal primarily with adjustments to the number of shares included in the index to account for share repurchases, seasoned equity offerings, and similar corporate events. For the Nasdaq 100 index, a larger fraction of constituent changes takes place on the quarterly rebalance dates than for the S&P 100 index, but a sizeable fraction does not.

[36] The data are available at http://www.nasdaq.com/indexshares/historical_data.stm.

Table 14: Specification analysis: Relationship between index and factor returns.

The table reports the coefficients of determination from regressions of the returns on the S&P 100 and Nasdaq 100 indices on the common return factors extracted from the panel of stock returns when using one, two and three common return factors. The sample period is January 1996 to October 2009.

Index	Number of Return Factors		
	1	2	3
S&P 100	91.78%	92.23%	92.05%
Nasdaq 100	71.33%	89.22%	89.23%

Time series of index vs. factor returns: In order to test whether the factors reproduce the time series of index returns, we first extract the realizations of the common return factors from our panel of asset returns using a standard factor analysis allowing for one, two, and three common factors, and then regress the returns of the two indices on these common factors.[37] Table 14 reports the coefficients of determination from these regressions. Observe that with a R^2 value of around 92%, a single factor accurately reproduces the returns on the S&P 100 index. However, a second factor is required to match the returns on the Nasdaq 100 index with comparable accuracy. Increasing the number of factors to three does not produce any material improvement in the R^2 value for the Nasdaq 100 index and even causes a slight decrease in R^2 for the S&P 100 index.[38] We conclude that a model with two common return factors is appropriate for our analysis.

[**Table 14 about here**]

Time series of index vs. factor variances: We now verify that the variances of the common return factors can account for index variances and their movement through time. When the number of common return factors is too small, residual returns will be correlated across assets. Denote by $\rho_{m,n,\epsilon,t}$ the correlation in residual returns between assets m and n. The variance of index returns $dI_{p,t}/I_{p,t}$ is given by

$$\sigma^2_{I,p,t} = \beta'_{I,p,t}\Sigma_t\beta_{I,p,t} + \sum_{n=1}^{N}\sum_{m\neq n}^{N} w_{m,p,t}w_{n,p,t}\rho_{m,n,\epsilon,t}\sqrt{V_{m,\epsilon,t}V_{n,\epsilon,t}} + \sum_{n=1}^{N} w^2_{n,p,t}V_{n,\epsilon,t} \ . \tag{E.1}$$

Thus, index variance consists of three components: (i) a component reflecting the variances and covariances of the common return factors Σ_t and assets' average factor exposures $\beta_{I,p,t}$, (ii) the weighted-average covariance between the return residuals, and (iii) the sum of the assets' idiosyncratic variances multiplied with the square of their weight in the index.[39]

A simple specification test is whether any significant covariances in residual returns manifest themselves in index variance. In order to assess the relative magnitude of the three variance components, we compute each using historical asset returns and the factor realizations obtained from the factor analysis of stock returns. For each date we compute the variances and covariances of the common factors Σ_t, the residual

[37] Sentana and Fiorentini (2001) and Sentana (2004) show that standard factor analysis can be used even in the presence of stochastic volatility. Specifically, Sentana (2004) shows that if the factor loadings are constant over time and the unconditional variances of common and idiosyncratic factors are constant, then the unconditional covariance matrix of return innovations will inherit the factor structure.

[38] Since the regressors are obtained from the factor analysis of asset returns, the first two regressors change when the third is added. This explains why the R^2 values can decrease when additional factors are added.

[39] Recall from Section 2.2 that our methodology allows adjusting for idiosyncratic variances when extracting the variance swap rates on the common return factors, so the magnitude of (iii) is not a concern. In addition, it turns out that (iii) is small.

Table 15: Specification analysis: Components of realized variance in the S&P 100 and Nasdaq 100.

The table reports summary statistics for realized index variance $\sigma^2_{I,p,t}$ and its components in the decomposition

$$\sigma^2_{I,p,t} = \beta'_{I,p,t}\Sigma_t\beta_{I,p,t} + \sum_{n=1}^{N}\sum_{m\neq n}^{N} w_{m,p,t}w_{n,p,t}\rho_{m,n,\epsilon,t}\sqrt{V_{m,\epsilon,t}V_{n,\epsilon,t}} + \sum_{n=1}^{N} w^2_{n,p,t}V_{n,\epsilon,t}\ ,$$

namely (i) $\beta'_{I,p,t}\Sigma_t\beta_{I,p,t}$, the component reflecting the variances and covariances of the common return factors Σ_t and assets' average factor exposures $\beta_{I,p,t}$, (ii) $\sum_{n=1}^{N}\sum_{m\neq n}^{N} w_{m,p,t}w_{n,p,t}\rho_{m,n,\epsilon,t}\sqrt{V_{m,\epsilon,t}V_{n,\epsilon,t}}$, the component reflecting the weighted-average covariance between the return residuals, and (iii) $\sum_{n=1}^{N} w^2_{n,p,t}V_{n,\epsilon,t}$, the component reflecting the sum of the assets' idiosyncratic variances multiplied with the square of their weight in the index. The sample period is January 1996 to October 2009.

	Mean	Std.	Correlation with		
			Cov.	Idio.	Index
A. S&P 100 Index					
Factors $\beta'_{I,p,t}\Sigma_t\beta_{I,p,t}$	0.0442	0.0723	36.95%	59.49%	**97.78%**
Residual covariances $\sum_{n=1}^{N}\sum_{m\neq n}^{N} w_m w_n \sigma_{m,n,\epsilon,t}$	0.0012	0.0019		55.94%	41.22%
Residual variances $\sum_{n=1}^{N} w^2_n V_{n,\epsilon,t}$	0.0018	0.0017			60.58%
Index variance $\sigma^2_{I,p,t}$	0.0458	0.0736			
B. Nasdaq 100 Index					
Factors $\beta'_{I,p,t}\Sigma_t\beta_{I,p,t}$	0.0979	0.1134	60.59%	56.40%	**95.96%**
Residual covariances $\sum_{n=1}^{N}\sum_{m\neq n}^{N} w_m w_n \sigma_{m,n,\epsilon,t}$	0.0054	0.0081		60.81%	69.81%
Residual variances $\sum_{n=1}^{N} w^2_n V_{n,\epsilon,t}$	0.0047	0.0041			62.86%
Index variance $\sigma^2_{I,p,t}$	0.1155	0.1435			

covariances $\sigma_{m,n,\epsilon,t} = \rho_{m,n,\epsilon,t}\sqrt{V_{m,\epsilon,t}V_{n,\epsilon,t}}$, and the residual variances $V_{n,\epsilon,t}$ using trailing one-month (21 trading day) windows. For each stock index, we apply the index weights on the last day of the estimation window in order to obtain the three components in (E.1) for that index.

[Table 15 and Figure 5 about here]

Table 15 provides summary statistics, and Figure 5 plots the three components of index variance as well as the total index variance for both indices over our sample period. The conclusion emerging from these results is striking: the variances of the two common return factors account for the overwhelming part of the level and the variability of index variances. As can be seen in Figure 5, the contribution of both the average covariance between the return residuals and the idiosyncratic return variances to index variances are extremely small. Even at their peak during the burst of the Internet bubble in the year 2000, the two components make up only a small share of overall index variance. In Table 15, the level of the factor component amounts to over 96% (84%) of index variance and its standard deviation to over 98% (79%) of the standard deviation of index variance for the S&P 100 and Nasdaq 100 indices, respectively. The average residual covariance amounts to less than 3% (5%) of index variance for the S&P 100 and Nasdaq 100 indices, respectively. Finally, the correlation between the factor component and index variance exceeds 95% for both indices. Thus, two common return factors are able to capture the realized variances of both indices and their movements through time.

Return Correlations: The final test is whether the common return factors can account for the correlation in asset returns and its movement through time. Figure 6 shows the average return correlation and the average residual correlation of the index constituents over the sample period when using two return factors. For both indices, the average return correlation among assets fluctuates significantly through time, but the average residual correlation is almost zero throughout the sample period. Thus, the two return factors accurately capture correlations in the returns of the index constituents and their variation through time. We conclude that a model with two return factors appears sufficiently accurate for our analysis.

[Figure 6 about here]

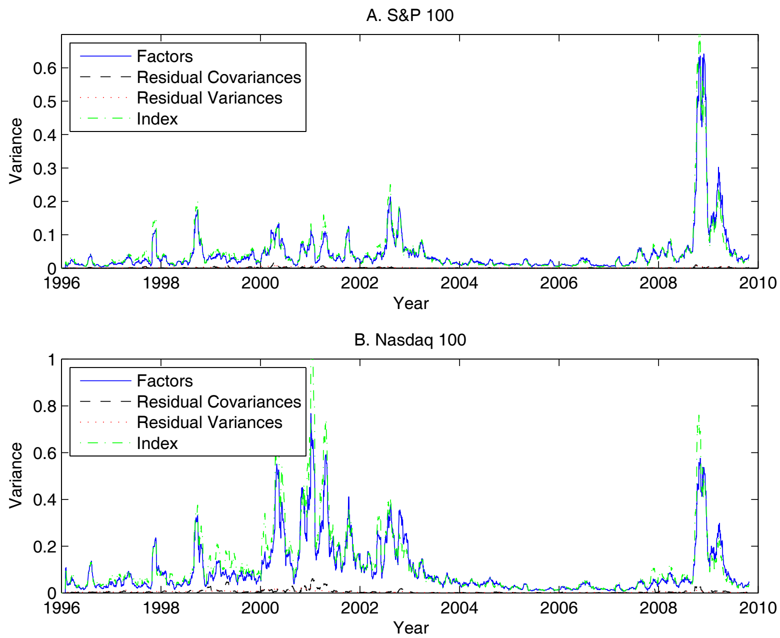

Figure 5: Specification analysis of the return factor model: Components of realized variance in the S&P 100 and Nasdaq 100 indices during the sample period.

Number of Return Factors vs. Number of Variance Factors: How is the two-factor structure in returns consistent with the cross-section of realized variances and variance swap rates documented in Section 3.2? To the extent that common factors in variance swap rates capture common components in assets' *idiosyncratic* variances, it is natural that the number of variance factors exceeds the number of return factors. The reason is that common components in assets' idiosyncratic variances explain the cross-section of asset variances, but do not contribute materially to explaining index variances. This nuance is important: although idiosyncratic returns are uncorrelated across assets, this does not rule out that their variances may be correlated. The results in Table 3 therefore suggest the presence of common factors in assets' idiosyncratic variances and variance swap rates. Table 15 also reveals that for both stock indices, the residual variance component is strongly correlated with the factor variance component, with correlations of 59.49% and 56.40% for the S&P 100 and Nasdaq 100 indices, respectively. This suggests that individual assets' idiosyncratic return variances are positively correlated with factor variances. We document this formally in Section 4.1.

F Optimal Return Factor Rotation and Factor Variance Swap Extraction

In this appendix we establish the optimality of the factor rotation that we use and provide the details of the implementation of our factor extraction methodology.

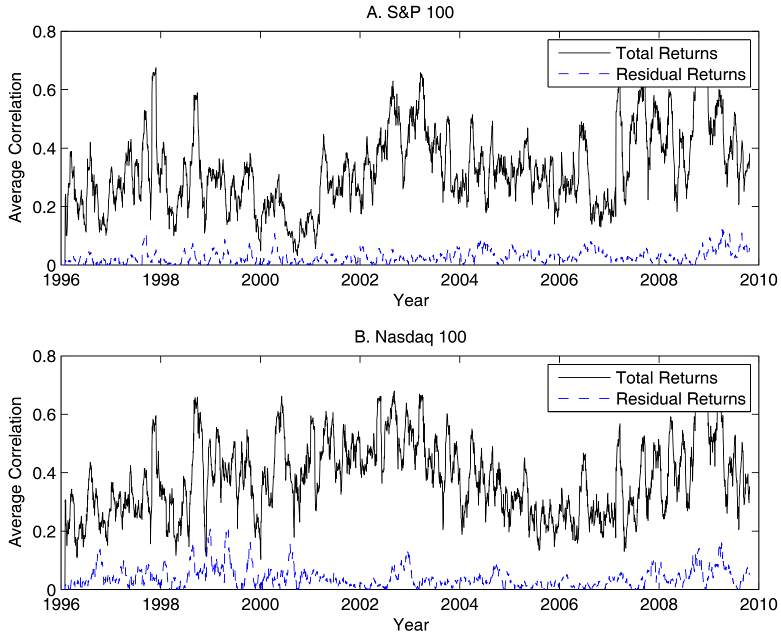

Figure 6: Specification analysis of the return factor model: Average return correlations and residual return correlations in the S&P 100 and Nasdaq 100 indices during the sample period.

Optimal Return Factor Rotation: Let $F_t = (F_{1t}, F_{2t})'$ denote the true factors and assume that their instantaneous variance-covariance matrix is $\Sigma_t = \begin{pmatrix} \sigma_{1t}^2 & 0 \\ 0 & \sigma_{2t}^2 \end{pmatrix}$. To simplify the exposition, suppose that the factors have been scaled to have unit average variance, i.e., $E(\sigma_{it}^2) = 1$. Let $\tilde{F}_t$ denote the factor estimates produced by a standard factor analysis; these estimates are a rotated version of the true factors. Letting θ denote the rotation angle when moving from the true factors to the estimated ones, the rotation matrix is $R = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}$. Hence, the instantaneous variance-covariance matrix of the estimated factors is

$$\tilde{\Sigma}_t = R'\Sigma_t R = \begin{pmatrix} \sigma_{1t}^2 \cos^2(\theta) + \sigma_{2t}^2 \sin^2(\theta) & (\sigma_{2t}^2 - \sigma_{1t}^2)\sin(\theta)\cos(\theta) \\ (\sigma_{2t}^2 - \sigma_{1t}^2)\sin(\theta)\cos(\theta) & \sigma_{1t}^2 \sin^2(\theta) + \sigma_{2t}^2 \cos^2(\theta) \end{pmatrix}.$$

Note that the expectation of this matrix is the identity matrix regardless of the rotation angle, so it cannot be used to identify θ. However, the variance of the off-diagonal elements contains the necessary information provided that the variances of the two factors σ_{1t}^2 and σ_{2t}^2 do not move perfectly in sync. Indeed, one has

$$\mathrm{var}((\sigma_{2t}^2 - \sigma_{1t}^2)\sin(\theta)\cos(\theta)) = \sin^2(\theta)\cos^2(\theta)\mathrm{var}(\sigma_{2t}^2 - \sigma_{1t}^2),$$

which is minimized when $\theta = 0$ or $\theta = \pi/2$, i.e. when the estimated factors are equal to the true factors up to permutation. Hence, we can identify the angle $\tilde{\theta}$ by which we need to rotate the estimated factors to obtain the true factors by finding the value of $\tilde{\theta}$ for which the time series standard deviation of a local measure of the covariance between the rotated estimated factors is minimized. In our empirical implementation, we use local covariance estimates computed over one-month (21 trading day) windows.

[Table 16 about here]

Table 16 reports the correlation coefficients between the optimally rotated return factors and the Fama-French and Carhart factors. The first rotated return factor is strongly correlated with the market return and close to uncorrelated with the HML factor. The second return factor is somewhat correlated with the market and strongly negatively correlated with HML. Both factors are slightly correlated with SMB and UMD.

Table 16: Specification analysis: Correlation between common return factors and $FF4$ factors.

This table reports the correlations between the common factors extracted from the panel of asset returns and rotated such that the time series standard deviation of the factor covariances estimated over 21-day periods is minimized and the Fama-French and Carhart factors. The sample period is January 1996 to October 2009.

Factor	Market	Size (SMB)	Price/Book (HML)	Momentum (UMD)
Factor 1	84.89%	−24.71%	11.81%	−37.92%
Factor 2	43.77%	25.45%	−60.13%	−10.66%

Factor Variance Swap Rate Extraction: Dropping the covariance swap rate corresponds to dropping the B terms in the matrix X_t and the covariance swap rate in the vector $\Phi_{t,\tau}$ in (15). After factor rotation we compute the time series of the factor exposures for the S&P 100 and Nasdaq 100 index (the $A_{p,j,t}$ terms in (16)). For robustness we perform this task two ways—with constant or time-varying betas: In the basic specification, we assume that the assets' factor exposures are constant through time and estimate β_n using OLS by regressing the log returns $r_{n,t}$ on the rotated factor scores F_t. Alternatively, we allow the assets' factor exposures to be time-varying. We assume they follow a random walk, $\beta_{n,t} = \beta_{n,t-1} + \eta_{n,t}$, and estimate $\beta_{n,t}$ using the Kalman filter based on the measurement equation $r_{n,t} = \beta'_{n,t}F_t + \epsilon_{n,t}$. We then combine the assets' estimated factor exposures obtained using each approach with the index weights to compute $A_{p,j,t}$ from (18).

With the time series of adjusted factor exposure matrices X_t in hand, we compute the factor variance swap rates from the adjusted index variance swap rates $y_{p,t}$ with the Kalman filter, using (15) as the measurement equation and specifying that the factor variance swap rates $VS^{ii}_{t,\tau} = \Phi_{t,\tau}(i)$ follow mean-reverting processes

$$\Phi_{t,\tau}(i) = \kappa_i(\overline{\Phi}(i) - \Phi_{t,\tau}(i)) + \sigma_i\sqrt{\Phi_{F,t}(i)}\zeta_{i,t} \;, i \in \{1, 2\} \tag{F.1}$$

where $\zeta_{i,t}$ is noise.

Empirical Estimates from Factor Variance Swap Rate Extraction: Figure 3 illustrates the results from each step in the factor variance swap rate extraction. The left panels present the case of constant asset factor exposures, the right panels the case of time-varying factor exposures. The top panels show the adjusted factor exposures of the S&P 100 and Nasdaq 100 indices (the $A_{p,j,t}$ terms in (18)). Both indices have similar exposures to the first factor, but very different exposures to the second. This is expected as the first factor loads heavily on the market factor and the second on the price/book factor (see Table 16). Even though individual assets' factor exposures in the left panel are constant by assumption, both indices' factor exposures vary significantly through time because of changes in index constituents and weights. The middle panels in Figure 3 report the adjusted index variance swap rates (the $y_{p,t}$ terms in (15)); these are the same in the left and right panels. The bottom panels show the time series of the two factor variance swap rates estimated using the Kalman filter. Importantly, the assumption of constant or time-varying factor exposures has only a minor impact on the factor variance swap rate estimates; the correlation of the factor variance swap rates obtained using the two approaches is 97.89% for the first factor and 94.74% for the second. Although they are very similar, we report both the results obtained assuming constant factor exposures and those obtained using time-varying factor exposures.

The results in the bottom panels of Figure 3 reveal the importance of allowing for two factor variance swap rates. Indeed, the correlation between the two factor variance swap rates is only 10%, and their peaks do not occur concurrently. For instance, the peaks in index variance swap rates that occurred during the 1997 Asia financial crisis, the 1998 financial crisis, September 2001, the 2002-2003 recession and the 2008-2009 financial crisis are all primarily driven by the first factor, while the extremely large variance swap rates on the Nasdaq 100 index during the burst of the Internet bubble in the years 2000 and 2001 are mostly driven by the second factor.

G A Simple Model of Variance Risk Pricing

In this appendix, we present a simple model formalizing the intuition provided in the text on the determinants of variance risk premia in the presence of demand or supply pressure from end users of index and single-stock options. Consider a representative option market maker that has capital W_t. For simplicity, assume that the market maker trades variance swaps rather than individual options; this assumption greatly simplifies the derivations, while capturing the essential feature that buying or selling options and delta-hedging them leaves the market maker exposed to variance shocks. In other words, the problem we solve here is the one that the market maker would be facing assuming that he delta-hedges his option positions.

For simplicity, we consider the case with a single return factor and a single common idiosyncratic variance factor. Hence, using the notation from Section 1, the instantaneous variance of returns on each stock $n = 1, \ldots, N$ is given by

$$\sigma_{n,t}^2 = \beta_{n,t}^2 \Sigma_t + \gamma_{n,t} \Gamma_t + \tilde{V}_{n,\epsilon,t} \ , \tag{G.1}$$

where Σ_t denotes the variance of the common return factor, Γ_t the common idiosyncratic variance factor, and $\tilde{V}_{n,\epsilon,t}$ stock n's truly idiosyncratic return variance. Similarly, letting $w_{n,t}$ denote stock n's weight in the index, the instantaneous variance of index returns is given by

$$\sigma_{I,t}^2 = \beta_{I,t}^2 \Sigma_t + \gamma_{I,t} \Gamma_t + \sum_{n=1}^{N} w_{n,t}^2 \tilde{V}_{n,\epsilon,t} \ , \tag{G.2}$$

where $\beta_{I,t} = \sum_n w_{n,t} \beta_{n,t}$ denotes the index's exposure to the return factor, $\gamma_{I,t} = \sum_n w_{n,t}^2 \gamma_{n,t}$ its exposure to common idiosyncratic variance shocks, and $\sum_{n=1}^{N} w_{n,t}^2 \tilde{V}_{n,\epsilon,t}$ index variance resulting from stocks' truly idiosyncratic variances. The last two terms in (G.2) are small if the index is well-balanced; we shall however consider them in the analysis for completeness.

We assume that the variance of the common return factor Σ_t, the common idiosyncratic variance factor Γ_t and the N truly idiosyncratic return variances $\tilde{V}_{n,\epsilon,t}$, $n = 1, \ldots, N$ follow diffusion processes

$$\begin{aligned} d\Sigma_t &= \mu_{\Sigma,t} dt + \sigma_{\Sigma,t} dB_{\Sigma,t} \ , \\ d\Gamma_t &= \mu_{\Gamma,t} dt + \sigma_{\Gamma,t} dB_{\Gamma,t} + \sigma_{\Gamma\Sigma,t} dB_{\Sigma,t} \ , \\ d\tilde{V}_{n,\epsilon,t} &= \mu_{n,t} dt + \sigma_{n,t} dB_{n,t} \ . \end{aligned} \tag{G.3}$$

Consistent with the empirical evidence in Section 4.1, we allow factor variance Σ_t and common idiosyncratic variance Γ_t to be correlated. Consistent with the fact that $\tilde{V}_{n,\epsilon,t}$ are truly idiosyncratic variances, we assume that for all assets $n = 1, \ldots, N$, $dB_{n,t}$ is independent of all other sources of uncertainty.

The market maker takes positions in the riskless asset, the variance swap on the stock market index, and the N individual stock variance swaps in order to maximize his expected utility of terminal wealth. The rate of return on the riskless asset is r_t, the dollar return on a notional investment of \$1 in the index variance swap

$$r_{I,t} = \beta_{I,t}^2 r_{\Sigma,t} + \gamma_{I,t} r_{\Gamma,t} + \sum_{n=1}^{N} w_{n,t}^2 r_{\epsilon,n,t} \tag{G.4}$$

and that on a notional investment of \$1 in the individual asset variance swaps

$$r_{n,t} = \beta_{n,t}^2 r_{\Sigma,t} + \gamma_{n,t} r_{\Gamma,t} + r_{\epsilon,n,t} \ , \tag{G.5}$$

where $r_{\Sigma,t} = \phi_{\Sigma,t} dt + \psi_{\Sigma,t} dB_{\Sigma,t}$ denotes the return on the factor variance swap, $r_{\Gamma,t} = \phi_{\Gamma,t} dt + \psi_{\Gamma,t} dB_{\Gamma,t} +$

$\psi_{\Gamma\Sigma,t} dB_{\Sigma,t}$ that on common idiosyncratic variance, and $r_{\epsilon,n,t} = \phi_{\epsilon,n,t} dt + \psi_{\epsilon,n,t} dB_{n,t}$ that on truly idiosyncratic variance.

To simplify matters, assume that at any time t, the market maker settles any open variance swap position from the previous instant (causing him to realize a gain or loss) and initiates new variance swap positions which cost zero to enter (see Egloff, Leippold and Wu (2009)). Thus, the market maker has a 100% cash position. Letting x denote the notional investment in the index variance swap and $\mathbf{y}$ the notional investments in the individual asset variance swaps normalized by total wealth, wealth dynamics are given by

$$\begin{aligned} \frac{dW_t}{W_t} &= r_t dt + x\left(\beta_{I,t}^2(\phi_{\Sigma,t} dt + \psi_{\Sigma,t} dB_{\Sigma,t}) + \gamma_{I,t}(\phi_{\Gamma,t} dt + \psi_{\Gamma,t} dB_{\Gamma,t} + \psi_{\Gamma\Sigma,t} dB_{\Sigma,t}) + \mathbf{z}_t'(\phi_{\epsilon,t} dt + \mathrm{diag}(\psi_{\epsilon,t}) d\mathbf{B}_t)\right) \\ &\quad + \mathbf{y}'\left(\mathbf{b}_t(\phi_{\Sigma,t} dt + \psi_{\Sigma,t} dB_{\Sigma,t}) + \gamma_t(\phi_{\Gamma,t} dt + \psi_{\Gamma,t} dB_{\Gamma,t} + \psi_{\Gamma\Sigma,t} dB_{\Sigma,t}) + (\phi_{\epsilon,t} dt + \mathrm{diag}(\psi_{\epsilon,t}) d\mathbf{B}_t)\right) \ , \end{aligned} \tag{G.6}$$

where variables where n subscripts have been dropped represent column vectors of previously subscripted variables, $\mathbf{z}_t \equiv \mathbf{w}_t \odot \mathbf{w}_t$ and $\mathbf{b}_t \equiv \beta_t \odot \beta_t$, where $\odot$ denotes the Hadamard product. Hence,

$$\begin{aligned} \left(\frac{dW_t}{W_t}\right)^2 &= \left(x\left(\beta_{I,t}^2 \psi_{\Sigma,t} + \gamma_{I,t} \psi_{\Gamma\Sigma,t}\right) + \mathbf{y}'\left(\mathbf{b}_t \psi_{\Sigma,t} + \gamma_t \psi_{\Gamma\Sigma,t}\right)\right)^2 dt \\ &\quad + \left(x\gamma_{I,t} + \mathbf{y}'\gamma_t\right)^2 \psi_{\Gamma,t}^2 dt + (\mathbf{z}_t' + \mathbf{y}')\,\mathrm{diag}(\psi_{\epsilon,t})^2 (\mathbf{z}_t + \mathbf{y})\, dt \ . \end{aligned} \tag{G.7}$$

Letting $J(W,t)$ denote the indirect utility of wealth, the Bellman equation is

$$\begin{aligned} 0 &= \max_{x,\mathbf{y}} J_t + J_W W_t \left(r_t + x\left(\beta_{I,t}^2 \phi_{\Sigma,t} + \gamma_{I,t}\phi_{\Gamma,t} + \mathbf{z}_t'\phi_{\epsilon,t}\right) + \mathbf{y}'\left(\mathbf{b}_t \phi_{\Sigma,t} + \gamma_t \phi_{\Gamma,t} + \phi_{\epsilon,t}\right)\right) \\ &\quad + \frac{1}{2} J_{WW} W_t^2 \begin{pmatrix} \left(x\left(\beta_{I,t}^2 \psi_{\Sigma,t} + \gamma_{I,t}\psi_{\Gamma\Sigma,t}\right) + \mathbf{y}'\left(\mathbf{b}_t \psi_{\Sigma,t} + \gamma_t \psi_{\Gamma\Sigma,t}\right)\right)^2 \\ + \left(x\gamma_{I,t} + \mathbf{y}'\gamma_t\right)^2 \psi_{\Gamma,t}^2 + (\mathbf{z}_t' + \mathbf{y}')\,\mathrm{diag}(\psi_{\epsilon,t})^2 (\mathbf{z}_t + \mathbf{y}) \end{pmatrix} \ . \end{aligned} \tag{G.8}$$

The first-order optimality conditions for x and $\mathbf{y}$ are

$$\begin{aligned} 0 &= J_W W_t \left(\beta_{I,t}^2 \phi_{\Sigma,t} + \gamma_{I,t}\phi_{\Gamma,t} + \mathbf{z}_t'\phi_{\epsilon,t}\right) \\ &\quad + J_{WW} W_t^2 \begin{pmatrix} \left(\beta_{I,t}^2 \psi_{\Sigma,t} + \gamma_{I,t}\psi_{\Gamma\Sigma,t}\right)\left(x\left(\beta_{I,t}^2 \psi_{\Sigma,t} + \gamma_{I,t}\psi_{\Gamma\Sigma,t}\right) + \mathbf{y}'\left(\mathbf{b}_t \psi_{\Sigma,t} + \gamma_t \psi_{\Gamma\Sigma,t}\right)\right) \\ + \gamma_{I,t}\left(x\gamma_{I,t} + \mathbf{y}'\gamma_t\right)\psi_{\Gamma,t}^2 \end{pmatrix} \end{aligned} \tag{G.9}$$

and

$$\begin{aligned} 0 &= J_W W_t \left(\mathbf{b}_t \phi_{\Sigma,t} + \gamma_t \phi_{\Gamma,t} + \phi_{\epsilon,t}\right) \\ &\quad + J_{WW} W_t^2 \begin{pmatrix} \left(\mathbf{b}_t \psi_{\Sigma,t} + \gamma_t \psi_{\Gamma\Sigma,t}\right)\left(x\left(\beta_{I,t}^2 \psi_{\Sigma,t} + \gamma_{I,t}\psi_{\Gamma\Sigma,t}\right) + \mathbf{y}'\left(\mathbf{b}_t \psi_{\Sigma,t} + \gamma_t \psi_{\Gamma\Sigma,t}\right)\right) \\ + \gamma_t\left(x\gamma_{I,t} + \mathbf{y}'\gamma_t\right)\psi_{\Gamma,t}^2 + \mathrm{diag}(\psi_{\epsilon,t})^2 (\mathbf{z}_t + \mathbf{y}) \end{pmatrix} \ . \end{aligned} \tag{G.10}$$

To determine the equilibrium variance risk premia, note that market makers' net demand must equal non market makers' net supply. In other words, given net demand of index variance swaps of $-x$ and net demand for individual stock variance swaps of $-\mathbf{y}$ by non market makers, the equilibrium variance risk premia on the index and the individual assets is given by

$$\phi_{I,t} = \beta_{I,t}^2 \phi_{\Sigma,t} + \gamma_{I,t}\phi_{\Gamma,t} + \mathbf{z}_t'\phi_{\epsilon,t} \tag{G.11}$$

$$= \frac{-J_{WW} W}{J_W} \begin{pmatrix} \left(\beta_{I,t}^2 \psi_{\Sigma,t} + \gamma_{I,t}\psi_{\Gamma\Sigma,t}\right)\left(x\left(\beta_{I,t}^2 \psi_{\Sigma,t} + \gamma_{I,t}\psi_{\Gamma\Sigma,t}\right) + \mathbf{y}'\left(\mathbf{b}_t \psi_{\Sigma,t} + \gamma_t \psi_{\Gamma\Sigma,t}\right)\right) \\ + \gamma_{I,t}\left(x\gamma_{I,t} + \mathbf{y}'\gamma_t\right)\psi_{\Gamma,t}^2 \end{pmatrix} \tag{G.12}$$

and

$$\phi_t = \mathbf{b}_t \phi_{\Sigma,t} + \gamma_t \phi_{\Gamma,t} + \phi_{\epsilon,t} \tag{G.13}$$

$$= \frac{-J_{WW} W}{J_W} \begin{pmatrix} \left(\mathbf{b}_t \psi_{\Sigma,t} + \gamma_t \psi_{\Gamma\Sigma,t}\right)\left(x\left(\beta_{I,t}^2 \psi_{\Sigma,t} + \gamma_{I,t}\psi_{\Gamma\Sigma,t}\right) + \mathbf{y}'\left(\mathbf{b}_t \psi_{\Sigma,t} + \gamma_t \psi_{\Gamma\Sigma,t}\right)\right) \\ + \gamma_t\left(x\gamma_{I,t} + \mathbf{y}'\gamma_t\right)\psi_{\Gamma,t}^2 + \mathrm{diag}(\psi_{\epsilon,t})^2 (\mathbf{z}_t + \mathbf{y}) \end{pmatrix} \ . \tag{G.14}$$

For ease of interpretation, it is best to consider these expressions in the case $\gamma_{I,t} = 0$ and $\mathbf{z}_t = \mathbf{0}$; as mentioned above, these two components will be very small in practice. Then, the variance risk premia are

$$\phi_{I,t} = \frac{-J_{WW}W}{J_W}\left(x\beta_{I,t}^2 + \mathbf{y}'\left(\mathbf{b}_t + \gamma_t \frac{\psi_{\Gamma\Sigma,t}}{\psi_{\Sigma,t}}\right)\right)\beta_{I,t}^2\psi_{\Sigma,t}^2 \tag{G.15}$$

and

$$\begin{aligned}\phi_t &= \frac{-J_{WW}W}{J_W}\left(\begin{array}{c}(\mathbf{b}_t\psi_{\Sigma,t} + \gamma_t\psi_{\Gamma\Sigma,t})\left(x\beta_{I,t}^2\psi_{\Sigma,t} + \mathbf{y}'\left(\mathbf{b}_t\psi_{\Sigma,t} + \gamma_t\psi_{\Gamma\Sigma,t}\right)\right) \\ +\gamma_t\left(\mathbf{y}'\gamma_t\right)\psi_{\Gamma,t}^2 + \mathrm{diag}(\psi_{\epsilon,t})^2\mathbf{y}\end{array}\right) \\ &= \left(\mathbf{b}_t + \gamma_t\frac{\psi_{\Gamma\Sigma,t}}{\psi_{\Sigma,t}}\right)\frac{\phi_{I,t}}{\beta_{I,t}^2} + \frac{-J_{WW}W}{J_W}\left(\gamma_t\left(\mathbf{y}'\gamma_t\right)\psi_{\Gamma,t}^2 + \mathrm{diag}(\psi_{\epsilon,t})^2\mathbf{y}\right)\ . \end{aligned}\tag{G.16}$$

The first expression says that the index variance risk premium equals the product of the market maker's risk aversion, his net exposure to factor variance (accounting for both his index and his single-stock variance position) $x\beta_{I,t}^2 + \mathbf{y}'\left(\mathbf{b}_t + \gamma_t\frac{\psi_{\Gamma\Sigma,t}}{\psi_{\Sigma,t}}\right)$, the index's exposure to factor variance $\beta_{I,t}^2$, and the riskiness of factor variance $\psi_{\Sigma,t}^2$. The second expression says that variance risk premia on individual stocks are driven by their exposure to factor variance, plus the market maker's risk aversion multiplied with the sum of two components: (1) the assets' exposure to common idiosyncratic variance γ_t, times market maker's total exposure to common idiosyncratic variance $\mathbf{y}'\gamma_t$, times the riskiness of common idiosyncratic variance, $\psi_{\Gamma,t}^2$, and (2) the assets' contribution to the market maker's bearing of diversifiable variance risk, which is simply the product of the riskiness of truly idiosyncratic variance for each asset, $\psi_{\epsilon,n,t}^2$, and the net supply of variance for that asset, y_n.

The empirical implications of the model are the following:

1. Assuming, consistent with the empirical evidence, that end users' net supply of single-stock options requires market makers to be net long common idiosyncratic variance, i.e., $\mathbf{y}'\gamma_t > 0$, common idiosyncratic variance will command a positive risk premium. In other words, the cross-section of assets' variance risk premia will reflect their exposure to a common idiosyncratic variance factor, γ_t.
2. The price of risk for common idiosyncratic variance will be larger in periods where the total net supply of single-stock variance $\mathbf{y}'\gamma_t$ (i.e., the total net supply of single-stock options) is larger.
3. The price of risk for common idiosyncratic variance will be larger in periods where the riskiness of common idiosyncratic variance $\psi_{\Gamma,t}^2$ is larger.
4. An asset's variance risk premium will be greater, the greater the net supply of variance for that asset, y_n.
5. An asset's variance risk premium will be greater, the more variable truly idiosyncratic variance for that asset, $\psi_{\epsilon,n,t}^2$.

H Profitability of Dispersion Trading

In this section, we show that dispersion trading strategies earn a combination of systematic and idiosyncratic variance risk premia. Assume for simplicity that there is a single common return factor. Let $\overline{w}_{n,t} \geq 0$ be the weight of the variance swap on the nth stock in the dispersion trading portfolio and $\overline{w}_{I,p,t} \leq 0$ be the weight of the variance swap on index p when the dispersion trade is entered at time t. The excess return on the portfolio at time $t+\tau$ is

$$R_{t,\tau} = \sum_{n=1}^{N}\overline{w}_{n,t}\frac{\frac{1}{\tau}\int_t^{t+\tau}\sigma_{n,u}^2du - VS_{n,t,\tau}}{VS_{n,t,\tau}} + \overline{w}_{I,p,t}\frac{\frac{1}{\tau}\int_t^{t+\tau}\sigma_{I,p,u}^2du - VS_{I,p,t,\tau}}{VS_{I,p,t,\tau}}\ . \tag{H.1}$$

Substituting the asset and index variances (6) and variance swap rates (7) and (8) yields

$$\begin{aligned} R_{t,\tau} &= \left(\sum_{n=1}^{N} \frac{\overline{w}_{n,t}\beta_n^2}{VS_{n,t,\tau}} + \frac{\overline{w}_{I,p,t}\beta_{I,p,t}^2}{VS_{I,p,t,\tau}}\right)\left(\frac{1}{\tau}\int_t^{t+\tau} \Sigma_u du - VS_{t,\tau}\right) \\ &\quad + \sum_{n=1}^{N}\left(\left(\frac{\overline{w}_{n,t}}{VS_{n,t,\tau}} + \frac{\overline{w}_{I,p,t}w_{n,p,t}^2}{VS_{I,p,t,\tau}}\right)\left(\frac{1}{\tau}\int_t^{t+\tau} V_{n,\epsilon,u}du - VS_{n,\epsilon,t,\tau}\right)\right) \end{aligned} \tag{H.2}$$

Thus, the excess return of the dispersion trading strategy is a combination of the variance risk premium on the common return factor, $\frac{1}{\tau}\int_t^{t+\tau}\Sigma_u du - VS_{t,\tau}$, and of the idiosyncratic variance risk premia on the individual assets, $\frac{1}{\tau}\int_t^{t+\tau} V_{n,\epsilon,u}du - VS_{n,\epsilon,t,\tau}$.

The relative importance of these two components in the strategy's profitability depends on the weights of the individual asset and index variance swaps in the portfolio. By selecting the weights, one can construct portfolios that are exposed only to factor variance risk, only to idiosyncratic variance risk, or to both. For example, setting $\overline{w}_{I,p,t} = -1$ and letting $\overline{w}_{n,t} = w_{n,p,t}^2 VS_{n,t,\tau}/VS_{I,p,t,\tau}$ yields a portfolio that only earns the factor variance risk premium. Similarly, letting $\overline{w}_{n,t} = w_{n,p,t}$ and setting $\overline{w}_{I,p,t} = -\sum_{n=1}^{N} \frac{\overline{w}_{n,t}\beta_n^2}{VS_{n,t,\tau}}\frac{VS_{I,p,t,\tau}}{\beta_{I,p,t}^2}$ yields a portfolio that only earns the idiosyncratic variance risk premium. In general, dispersion trading strategies will earn a combination of both.[40]

[40]This is even the case for the strategy where $\overline{w}_{I,p,t} = -1$ and the weights of the variance swaps on the individual assets are set to match the index weights $w_{n,p,t}$. In this case, (H.2) becomes

$$\begin{aligned} R_{t,\tau} = &\left(\sum_{n=1}^{N} \frac{w_{n,p,t}\beta_n^2}{VS_{n,t,\tau}} - \frac{\beta_{I,p,t}^2}{VS_{I,p,t,\tau}}\right)\left(\frac{1}{\tau}\int_t^{t+\tau} \Sigma_u du - VS_{t,\tau}\right) \\ &+ \sum_{n=1}^{N}\left(\left(\frac{w_{n,p,t}}{VS_{n,t,\tau}} - \frac{w_{n,p,t}^2}{VS_{I,p,t,\tau}}\right)\left(\frac{1}{\tau}\int_t^{t+\tau} V_{n,\epsilon,u}du - VS_{n,\epsilon,t,\tau}\right)\right). \end{aligned} \tag{H.3}$$

Observe that using (7) and (8), the term in the first bracket can be rewritten as $\frac{1}{VS_{I,p,t,\tau}}[(\sum_{n=1}^{N} w_{n,p,t}^2 VS_{n,\epsilon,t,\tau})\sum_{n=1}^{N}\frac{w_{n,p,t}\beta_n^2}{VS_{n,t,\tau}} - \beta_I^2\sum_{n=1}^{N}\frac{w_{n,p,t}VS_{n,\epsilon,t,\tau}}{VS_{n,t,\tau}}]$, which will typically be negative, while the term in the first bracket in the second summand will typically be positive (unless $w_{n,p,t} > VS_{I,p,t,\tau}/VS_{n,t,\tau}$, which is unlikely to occur for reasonably broad indices).

A portfolio constructed by buying a number of variance swaps on each asset that is proportional to this asset's weight in the index also earns a combination of factor and idiosyncratic variance risk premia (here, both the weight of the factor variance swap and those of the idiosyncratic variance swaps are unambiguously positive). In this case, $\overline{w}_{I,p,t} = -1/VS_{I,p,t,\tau}$, $\overline{w}_{n,t} = w_{n,p,t}/VS_{n,t,\tau}$, and (H.2) becomes

$$R_{t,\tau} = \left(\sum_{n=1}^{N} w_{n,p,t}\beta_n^2 - \beta_{I,p,t}^2\right)\left(\frac{1}{\tau}\int_t^{t+\tau}\Sigma_u du - VS_{t,\tau}\right) + \sum_{n=1}^{N}\left(w_{n,p,t}(1-w_{n,p,t})\left(\frac{1}{\tau}\int_t^{t+\tau} V_{n,\epsilon,u}du - VS_{n,\epsilon,t,\tau}\right)\right).$$

References

Aït-Sahalia, Yacine, and Robert Kimmel (2007): "Maximum Likelihood Estimation of Stochastic Volatility Models," *Journal of Financial Economics* 83, 413-452.

Andersen, Torben G., Tim Bollerslev, Francis X. Diebold, and Heiko Ebens (2001): "The Distribution of Realized Stock Return Volatility," *Journal of Financial Economics* 61, 43-76.

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang (2006): "The Cross-Section of Volatility and Expected Returns," *Journal of Finance* 61, 259-299.

Black, Fisher (1976): "Studies of Stock Price Volatility Changes," Proceeding of the 1976 meetings of the American Statistical Association, Business and Economical Statistics Section, 177-181.

Britten-Jones, Mark, and Anthony Neuberger (2000): "Option Prices, Implied Price Processes, and Stochastic Volatility," *Journal of Finance* 55, 839-866.

Broadie, Mark and Ashish Jain (2007): "Discrete and continuously sampled volatility and variance swaps," Working paper, Columbia Business School.

Buraschi, Andrea, Paolo Porchia, and Fabio Trojani (2010): "Correlation Risk and Optimal Portfolio Choice," *Journal of Finance* 65, 393-420.

Campbell, John Y. (1993): "Intertemporal Asset Pricing Without Consumption Data," *American Economic Review* 83, 487-512.

Campbell, John Y., Martin Lettau, Burton G. Malkiel, and Yexiao Xu (2001): "Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk, " *Journal of Finance* 56, 1-43.

Carr, Peter, and Dilip Madan (1998): "Towards a Theory of Volatility Trading," in: *Volatility*, Risk Publications, Robert Jarrow, ed., pp. 417-427.

Carr, Peter, and Liuren Wu (2009): "Variance Risk Premiums,"*Review of Financial Studies* 22, 1311-1341.

Chicago Board Options Exchange (CBOE) (2001): "Mutual Funds & Listed Options: Portfolio Management Strategies,"CBOE Investor Series Paper No. 1.

DaFonseca, José, Martino Grasselli, and Florian Ielpo (2009): "Hedging (Co)Variance Risk with Variance Swaps," Working Paper.

Demeterfi, Kresimir, Emmanuel Derman, Michael Kamal, and Joseph Zou (1999): "More Than You Ever Wanted To Know About Volatility Swaps," Goldman Sachs Quantitative Strategies.

Di Pietro, Vadim, and Gregory Vainberg (2006): "Systematic Variance Risk and Firm Characteristics in the Equity Options Market," Working Paper.

Driessen, Joost P., Pascal Maenhout, and Grigory Vilkov (2009): "The Price of Correlation Risk: Evidence from Equity Options,"*Journal of Finance* 64, 1377-1406.

Egloff, Daniel, Markus Leippold, and Liuren Wu (2009): "Variance Risk Dynamics, Variance Risk Premia, and Optimal Variance Swap Investments," Working Paper.

Erb, Claude B., Campbell R. Harvey, and Tadas E. Viskanta (1994): "Forecasting International Equity Correlations," Financial Analysts Journal, November/December, 32-45.

Fama, Eugene F., and Kenneth R. French (1992): "The Cross Section of Expected Stock Returns," *Journal of Finance* 47, 427-466.

Garleanu, Nicolae, Lasse H. Pedersen, and Allen M. Poteshman (2009): "Demand-Based Option Pricing," *Review of Financial Studies* 22, 4259-4299.

Goyal, Amit, and Alessio Saretto (2009): "Cross-Section of Option Returns and Volatility," *Journal of Financial Economics* 94, 310-326.

Jiang, George J., and Yisong S. Tian (2005): "The Model-Free Implied Volatility and Its Information Content," *Review of Financial Studies* 18, 1305-1342.

Lakonishok, Josef, Inmoo Lee, Neil D. Pearson, and Allen M. Poteshman (2007): "Option Market Activity," *Review of Financial Studies* 20, 813-857.

Longin, François, and Bruno Solnik (2001): "Extreme Correlation of International Equity Markets," *Journal of Finance* 56, 649-675.

Malliaris, Steven, and Hongjun Yan (2010): "Reputation Concerns and Slow-Moving Capital," Working Paper, Yale School of Management.

Mitton, Todd, and Keith Vorkink (2004): "Equilibrium under-diversification and the preference for skewness," *Review of Financial Studies* 20, 1255-1288.

Scott, Richard C., and Philip A. Horvath (1980): "On the Direction of Preference For Moments of Higher Order Than The Variance," *Journal of Finance* 35, 915919.

Sentana, Enrique (2004): "Factor representing portfolios in large asset markets," *Journal of Econometrics* 119, 257-289.

Sentana, Enrique, and Gabriele Fiorentini (2001): "Identification, estimation and testing of conditionally heteroskedastic factor models," *Journal of Econometrics* 102, 143-164.

Torné, Olaf (2010): "More accurate pricing of variance products," Working Paper.

Vilkov, Grigory (2008): "Variance Risk Premium Demystified," Working Paper.

Table 1: Relationship between index variances, constituent variances, and constituent correlations.

The table reports the correlations between realized index variance, the average realized variance of the index constituents, the average correlation between index constituents, and the product of the average realized variance and the average correlation for the S&P 100 and Nasdaq 100 indices. All series are computed using trailing 1 month (21 trading day) windows, and the averages are based on the index weights. The sample period is January 1996 to October 2009.

	Correlation with		
	Average Variance	Average Correlation	Product
	A. S&P 100 Index		
Index Variance	92.56%	54.13%	98.12%
Average Variance		33.22%	91.51%
Average Correlation			54.15%
	B. Nasdaq 100 Index		
Index Variance	96.85%	61.09%	99.09%
Average Variance		50.19%	96.33%
Average Correlation			63.62%

Table 2: Returns on sort portfolios of single-stock variance swaps.

The table reports the monthly returns of equally-weighted variance swap portfolios constructed on the basis of the ratio of historical variance in the previous month to the variance swap rate at the end of the previous month, variance swap returns in the previous month, stocks' historical variance in the previous month, variance swap rate at the end of the previous month, and stocks' exposure to the S&P 100 and Nasdaq 100 index returns. To avoid any lookahead bias, only stocks that are members of one of the two indices as of the portfolio formation date are included in the analysis. The sample period is January 1996 to October 2009.

Portfolio	Mean	S.D.	Sharpe	Min	Max	Turnover
A. Sort on Historical Variance over Variance Swap Rate						
1 (low)	1.73	10.91	0.55	-25.01	56.04	0.72
2	3.25	12.89	0.87	-30.97	75.25	0.80
3	4.71	13.53	1.21	-23.11	75.10	0.79
4	7.14	19.40	1.27	-14.96	131.38	0.79
5 (high)	10.56	24.81	1.47	-16.31	129.26	0.77
$5-1$	8.84	19.02	1.61	-20.85	111.02	0.75
B. Sort on Variance Swap Return						
1 (low)	1.89	10.60	0.62	-24.84	59.62	0.72
2	3.55	12.47	0.99	-28.04	79.13	0.79
3	5.02	14.38	1.21	-18.59	80.29	0.80
4	6.91	18.04	1.33	-20.93	117.21	0.79
5 (high)	10.01	25.09	1.38	-17.98	163.06	0.76
$5-1$	8.12	19.40	1.45	-17.99	130.01	0.74
C. Sort on Historical Variance						
1 (low)	0.12	5.44	0.08	-15.93	43.71	0.42
2	1.29	8.47	0.53	-16.76	64.45	0.65
3	3.23	11.34	0.99	-17.53	83.51	0.68
4	6.84	17.73	1.34	-27.30	90.76	0.64
5 (high)	15.93	38.41	1.44	-33.09	217.97	0.45
$5-1$	15.81	35.35	1.55	-18.42	207.93	0.44
D. Sort on Variance Swap Rate						
1 (low)	0.64	5.37	0.41	-13.75	46.54	0.19
2	1.36	8.22	0.57	-14.82	66.67	0.39
3	2.96	10.92	0.94	-15.99	70.36	0.41
4	7.00	17.73	1.37	-21.46	105.65	0.40
5 (high)	15.49	39.13	1.37	-43.68	226.40	0.22
$5-1$	14.84	36.11	1.42	-29.94	216.72	0.21
E. Sort on Exposure to S&P 100 Returns						
1 (low)	2.10	9.42	0.77	-17.78	60.14	0.02
2	4.77	14.34	1.15	-22.56	92.75	0.03
3	4.19	14.07	1.03	-23.08	99.81	0.03
4	6.26	17.60	1.23	-18.78	98.00	0.02
5 (high)	9.87	24.54	1.39	-25.92	138.13	0.01
$5-1$	7.78	19.63	1.37	-23.49	123.38	0.02
F. Sort on Exposure to Nasdaq 100 Returns						
1 (low)	1.62	9.35	0.60	-20.02	68.86	0.02
2	3.53	12.22	1.00	-23.04	95.52	0.03
3	4.47	15.58	0.99	-20.64	94.68	0.03
4	7.38	20.81	1.23	-26.83	130.59	0.02
5 (high)	10.23	25.17	1.41	-22.15	171.29	0.02
$5-1$	8.61	22.03	1.35	-20.62	163.71	0.02

Table 3: Factor structure in individual asset realized variances, variance swap rates, and variance swap returns, and the relationship between common factors and index quantities.

The table reports the fraction of the variation explained by common factors extracted from the panel of individual assets. The second and third rows report the coefficient of determination from regressions of the index quantity for the S&P 100 and Nasdaq 100 indices on the common factors. Panel A reports the quantities for realized variances. Panels B and C report the quantities for variance swap rates and variance swap returns, respectively. The sample period is January 1996 to October 2009.

Asset or Index	Number of Factors					
	1	2	3	4	5	6
A. Realized Variances						
Individual assets	44.00%	55.16%	58.51%	61.91%	64.29%	65.43%
R^2 from S&P index on factors	78.64%	88.20%	92.02%	94.16%	94.31%	94.35%
R^2 from Nasdaq index on factors	57.88%	75.66%	75.66%	81.69%	84.07%	85.37%
B. Variance Swap Rates						
Individual assets	56.06%	70.44%	75.77%	77.92%	79.52%	81.31%
R^2 from S&P index on factors	74.94%	86.30%	88.36%	89.77%	90.82%	94.51%
R^2 from Nasdaq index on factors	76.33%	88.58%	91.13%	92.46%	95.32%	95.58%
C. Variance Swap Returns						
Individual assets	25.59%	28.57%	30.84%	31.83%	34.85%	35.72%
R^2 from S&P index on factors	68.11%	68.99%	76.48%	76.42%	76.93%	76.83%
R^2 from Nasdaq index on factors	66.10%	66.62%	67.45%	67.32%	74.53%	73.67%

Table 4: Correlation between variance swap components.

This table reports the correlations between variance swap rate components, namely the factor component in the index variance swap rate $\beta'_{I,p,t} VS_{t,\tau} \beta_{I,p,t}$, the idiosyncratic component in the index variance swap rate $\sum_n w^2_{n,p,t} VS_{n,\epsilon,t,\tau}$, the index variance swap rate $VS_{I,p,t,\tau}$, the variance swap rates on the two common return factors $VS^{11}_{t,\tau}$ and $VS^{22}_{t,\tau}$, the average variance swap rate of the index constituents $\sum_n w_{n,p,t} VS_{n,t,\tau}$, and the average idiosyncratic variance swap rate of the index constituents $\sum_n w_{n,p,t} VS_{n,\epsilon,t,\tau}$. Panels A and B report these correlations for the S&P 100 index assuming constant and, respectively, time-varying factor exposures. Panels C and D report them for the Nasdaq 100 index. The sample period is January 1996 to October 2009.

	Correlation with					
	$\sum_n w^2_{n,p,t} VS_{n,\epsilon}$	$VS_{I,p,t,\tau}$	$VS^{11}_{t,\tau}$	$VS^{22}_{t,\tau}$	$\sum_n w_{n,p,t} VS_{n,t,\tau}$	$\sum_n w_{n,p,t} VS_{n,\epsilon,t,\tau}$
A. S&P 100 Index, Constant Factor Exposures						
Factor component in index $\beta'_{I,p,t} VS_{t,\tau} \beta_{I,p,t}$	37.18%	99.78%	95.73%	28.19%	88.62%	52.99%
Idiosyncratic component in index $\sum_n w^2_{n,p,t} VS_{n,\epsilon,t,\tau}$		38.70%	17.05%	71.19%	73.71%	92.54%
Index variance swap $VS_{I,p,t,\tau}$			95.53%	28.79%	89.15%	54.37%
Factor variance swap 1 $VS^{11}_{t,\tau}$				1.73%	75.28%	36.99%
Factor variance swap 2 $VS^{22}_{t,\tau}$					56.66%	60.10%
Average variance swap $\sum_n w_{n,p,t} VS_{n,t,\tau}$						85.34%
B. S&P 100 Index, Time-Varying Factor Exposures						
Factor component in index $\beta'_{I,p,t} VS_{t,\tau} \beta_{I,p,t}$	2.45%	99.76%	92.94%	24.00%	88.33%	19.67%
Idiosyncratic component in index $\sum_n w^2_{n,p,t} VS_{n,\epsilon,t,\tau}$		4.85%	-13.03%	57.02%	44.34%	94.46%
Index variance swap $VS_{I,p,t,\tau}$			92.87%	25.30%	89.15%	22.08%
Factor variance swap 1 $VS^{11}_{t,\tau}$				-4.15%	74.82%	4.08%
Factor variance swap 2 $VS^{22}_{t,\tau}$					50.67%	57.44%
Average variance swap $\sum_n w_{n,p,t} VS_{n,t,\tau}$						60.27%
C. Nasdaq 100 Index, Constant Factor Exposures						
Factor component in index $\beta'_{I,p,t} VS_{t,\tau} \beta_{I,p,t}$	38.75%	99.94%	42.52%	89.88%	96.52%	74.23%
Idiosyncratic component in index $\sum_n w^2_{n,p,t} VS_{n,\epsilon,t,\tau}$		40.79%	23.84%	33.77%	51.57%	68.15%
Index variance swap $VS_{I,p,t,\tau}$			41.82%	90.21%	96.89%	75.31%
Factor variance swap 1 $VS^{11}_{t,\tau}$				1.73%	35.46%	20.42%
Factor variance swap 2 $VS^{22}_{t,\tau}$					88.64%	69.77%
Average variance swap $\sum_n w_{n,p,t} VS_{n,t,\tau}$						89.05%
D. Nasdaq 100 Index, Time-Varying Factor Exposures						
Factor component in index $\beta'_{I,p,t} VS_{t,\tau} \beta_{I,p,t}$	37.83%	99.81%	42.42%	82.87%	96.50%	68.57%
Idiosyncratic component in index $\sum_n w^2_{n,p,t} VS_{n,\epsilon,t,\tau}$		40.31%	7.75%	33.26%	50.45%	71.19%
Index variance swap $VS_{I,p,t,\tau}$			40.85%	83.59%	96.89%	70.12%
Factor variance swap 1 $VS^{11}_{t,\tau}$				-4.15%	34.61%	14.56%
Factor variance swap 2 $VS^{22}_{t,\tau}$					80.90%	55.31%
Average variance swap $\sum_n w_{n,p,t} VS_{n,t,\tau}$						84.46%

Table 5: Decomposition of variance risk premia.

This table reports monthly index-weighted average variance risk premia (VRP) for the index constituents of the S&P 100 and, respectively, Nasdaq 100 and the decomposition of total variance risk premia into systematic and idiosyncratic variance components for the entire sample period as well as split by calendar year. Following Carr and Wu (2009), we measure realized variance risk premia by the returns on short-dated variance swaps and compute variance swap returns over the period t to $t+\tau$ (τ is chosen to be one month) using the following decomposition on the n stocks:

$$\underbrace{\frac{RV_{n,t,\tau} - VS_{n,t,\tau}}{VS_{n,t,\tau}}}_{\text{Total variance risk premium}} = \underbrace{\frac{\beta'_{n,t}(RV_{t,\tau} - VS_{t,\tau})\beta_{n,t}}{VS_{n,t,\tau}}}_{\text{Systematic variance risk premium}} + \underbrace{\frac{RV_{n,\epsilon,t,\tau} - VS_{n,\epsilon,t,\tau}}{VS_{n,t,\tau}}}_{\text{Idiosyncratic variance risk premium}},$$

where $RV_{n,t,\tau}$ denotes the realized variance and $VS_{n,t,\tau}$ the variance swap rate on stock $n = 1, \ldots, N$. $RV_{t,\tau}$, $VS_{t,\tau}$, $RV_{n,\epsilon,t,\tau}$, and $VS_{n,\epsilon,t,\tau}$ are defined analogously. Newey-West t-statistics are reported in parentheses. The sample period is January 1996 to October 2009.

A. S&P 100 Members

			Specification 1: Constant factor exposures				*Specification 2*: Time-varying factor exposures			
	Total variance		Systematic variance		Idiosyncratic variance		Systematic variance		Idiosyncratic variance	
Year	VRP	t-stat	VRP	t-stat	VRP	t-stat	VRP	t-stat	VRP	t-stat
96-09	3.70	(1.06)	-19.73	(-8.19)	23.43	(14.28)	-24.03	(-8.19)	27.73	(19.78)
1996	2.14	(0.39)	-27.30	(-7.41)	29.44	(6.39)	-33.62	(-7.89)	35.75	(7.38)
1997	11.11	(1.39)	-25.73	(-3.75)	36.84	(12.11)	-29.48	(-3.85)	40.59	(13.16)
1998	24.63	(2.10)	-23.67	(-3.17)	48.30	(8.05)	-26.67	(-3.09)	51.30	(10.10)
1999	11.43	(1.85)	-20.04	(-8.79)	31.47	(6.19)	-22.70	(-6.84)	34.13	(7.87)
2000	34.48	(3.74)	3.07	(0.62)	31.41	(5.51)	10.18	(1.70)	24.31	(5.17)
2001	-3.25	(-0.29)	-18.07	(-2.58)	14.83	(2.24)	-27.39	(-3.08)	24.14	(3.96)
2002	16.47	(1.40)	-12.22	(-1.92)	28.70	(4.72)	-19.36	(-2.36)	35.84	(7.85)
2003	-25.57	(-11.48)	-41.62	(-20.34)	16.05	(7.58)	-55.36	(-22.47)	29.79	(12.21)
2004	-20.01	(-3.75)	-34.38	(-17.23)	14.38	(3.17)	-47.35	(-18.68)	27.34	(5.98)
2005	-19.41	(-4.66)	-27.06	(-13.84)	7.64	(2.66)	-33.07	(-13.28)	13.66	(5.23)
2006	-20.09	(-4.27)	-29.74	(-11.12)	9.65	(3.04)	-39.02	(-11.28)	18.93	(6.72)
2007	-4.16	(-0.54)	-12.69	(-1.94)	8.53	(3.24)	-20.11	(-2.81)	15.95	(8.35)
2008	55.43	(1.85)	24.61	(1.14)	30.82	(3.51)	32.34	(1.28)	23.10	(4.54)
2009	-17.25	(-2.24)	-35.44	(-9.40)	18.19	(4.03)	-25.33	(-4.41)	8.08	(2.96)

continued

Table 5: Decomposition of variance risk premia—continued.

B. Nasdaq 100 Members

			Specification 1: Constant factor exposures				*Specification 2*: Time-varying factor exposures			
	Total variance		Systematic variance		Idiosyncratic variance		Systematic variance		Idiosyncratic variance	
Year	VRP	t-stat	VRP	t-stat	VRP	t-stat	VRP	t-stat	VRP	t-stat
96-09	9.64	(3.21)	-11.99	(-6.00)	21.64	(12.99)	-12.42	(-5.61)	22.06	(15.85)
1996	-0.26	(-0.05)	-14.13	(-4.27)	13.87	(4.75)	-16.12	(-4.22)	15.86	(5.97)
1997	16.48	(2.15)	-11.81	(-2.06)	28.30	(7.03)	-9.67	(-1.57)	26.16	(7.83)
1998	33.58	(2.88)	-8.65	(-0.91)	42.23	(12.58)	-6.75	(-0.67)	40.33	(15.43)
1999	19.63	(3.90)	-22.64	(-8.87)	42.27	(8.76)	-21.28	(-7.53)	40.91	(8.92)
2000	56.76	(4.79)	-2.86	(-0.30)	59.62	(12.26)	6.13	(0.60)	50.63	(11.53)
2001	12.18	(1.28)	-6.60	(-1.12)	18.78	(4.35)	-5.22	(-0.75)	17.40	(4.77)
2002	16.09	(2.00)	-4.87	(-0.81)	20.96	(6.39)	-3.93	(-0.61)	20.02	(6.60)
2003	-16.91	(-6.58)	-26.33	(-10.68)	9.42	(6.23)	-30.81	(-12.69)	13.90	(8.89)
2004	-11.44	(-2.41)	-18.41	(-7.95)	6.97	(2.16)	-20.56	(-7.90)	9.13	(3.13)
2005	-8.45	(-1.57)	-15.40	(-6.91)	6.95	(1.42)	-20.20	(-7.15)	11.75	(2.44)
2006	-5.99	(-1.03)	-13.59	(-6.25)	7.60	(1.71)	-16.81	(-5.85)	10.82	(2.73)
2007	-1.90	(-0.29)	-13.32	(-3.74)	11.42	(2.53)	-20.43	(-5.17)	18.53	(4.83)
2008	37.70	(1.74)	14.66	(0.88)	23.04	(3.66)	16.02	(0.88)	21.68	(4.46)
2009	-20.36	(-3.49)	-28.15	(-7.79)	7.79	(2.83)	-28.29	(-6.39)	7.93	(3.95)

Table 6: Is common idiosyncratic variance risk a priced factor in the cross-section?

This table reports estimates of monthly factor risk premia in the cross-section of equity option/variance swap returns. Estimates are from a two-stage Fama-MacBeth procedure applied to the following linear model for expected excess returns on variance swaps:

$$E(r_{n,t,\tau} - r_{f,t}) = \sum_{i=1}^{4} \beta^i_{FF4}\lambda^i_{FF4} + \sum_{i=1}^{2} \beta^i_{SVR}\lambda^i_{SVR} + \sum_{i=1}^{2} \beta^i_{CIVR}\lambda^i_{CIVR},$$

where β denotes factor loadings and λ factor risk premia. $FF4$ are the four Fama-French factors, SVR are proxies for systematic variance risk factors (measured by S&P and, respectively, Nasdaq index variance swap returns), and $CIVR$ are proxies for common idiosyncratic variance risk factors (measured as the cross-sectional average variance swap return on the index constituents for each of the indices). t-statistics are reported in parentheses. The sample period is January 1996 to October 2009.

	λ	t-stat
Market	0.88	(10.31)
SMB	-1.37	(-7.95)
HML	0.26	(2.29)
MOM	-0.44	(-3.15)
SVR factor 1 (S&P)	-11.10	(-10.19)
SVR factor 2 (Nasdaq)	-0.37	(-0.34)
CIVR factor 1 (S&P)	12.59	(12.80)
CIVR factor 2 (Nasdaq)	13.30	(16.84)

Table 7: Can priced idiosyncratic variance risk explain the cross-section?

This table reports monthly abnormal portfolio returns (Alpha) and factor loadings for two expected return models, the Fama-French four-factor model ($FF4$) and the $FF4$ model augmented by variance risk factors ($FF4$+VR). The variance risk factors are as described in Table 6. Specifically, the two SVR factors are proxies for systematic variance risk (measured by S&P and, respectively, Nasdaq index variance swap returns), and the two $CIVR$ factors are proxies for common idiosyncratic variance risk (measured as the cross-sectional average variance swap return on the index constituents for each of the indices). The estimates are from time-series regressions for the 5-1 long-short sort portfolio returns constructed in Table 2. t-statistics are reported in parentheses. The sample period is January 1996 to October 2009.

Sort Variable	RV / VS		RV		VS		VRP		S&P β		Nasdaq β	
Model	$FF4$	$FF4$+VR	$FF4$	$FF4$+VR	$FF4$	$FF4$+VR	$FF4$	$FF4$+VR	$FF4$	$FF4$+VR	$FF4$	$FF4$+VR
Alpha	10.01	1.94	18.30	0.23	16.99	-2.56	9.00	1.28	8.70	0.82	8.92	0.53
	(6.81)	(1.17)	(6.97)	(0.10)	(6.25)	(-1.07)	(5.89)	(0.77)	(5.73)	(0.59)	(5.13)	(0.41)
Factor loadings:												
Market	-1.33	-0.09	-3.07	-0.26	-2.96	-0.26	-1.17	0.14	-1.22	-0.04	-0.78	-0.09
	(-3.93)	(-0.31)	(-5.09)	(-0.68)	(-4.73)	(-0.63)	(-3.32)	(0.51)	(-3.50)	(-0.19)	(-1.96)	(-0.38)
SMB	-0.16	0.51	-0.98	0.39	-0.70	0.62	0.00	0.65	-0.40	0.13	-0.38	-0.11
	(-0.40)	(1.68)	(-1.36)	(0.95)	(-0.93)	(1.41)	(0.00)	(2.13)	(-0.96)	(0.50)	(-0.79)	(-0.46)
HML	-0.94	-0.49	-1.56	-0.76	-1.10	-0.44	-0.59	-0.38	-0.30	-0.58	0.47	-0.68
	(-2.12)	(-1.38)	(-1.97)	(-1.60)	(-1.34)	(-0.85)	(-1.27)	(-1.07)	(-0.66)	(-1.93)	(0.91)	(-2.41)
MOM	-0.51	-0.35	-1.10	-0.83	-0.98	-0.71	-0.42	-0.30	-0.50	-0.43	-0.11	-0.11
	(-2.00)	(-1.81)	(-2.39)	(-3.27)	(-2.06)	(-2.57)	(-1.57)	(-1.56)	(-1.88)	(-2.69)	(-0.36)	(-0.75)
SVR factor 1	–	-0.13	–	0.22	–	-0.18	–	-0.09	–	-0.53	–	-1.73
	–	(-0.44)	–	(0.56)	–	(-0.41)	–	(-0.31)	–	(-2.15)	–	(-7.38)
SVR factor 2	–	0.99	–	1.92	–	2.00	–	1.17	–	1.49	–	2.05
	–	(6.08)	–	(8.83)	–	(8.48)	–	(7.18)	–	(10.89)	–	(15.81)
CIVR factor 1	–	2.11	–	3.86	–	4.03	–	1.53	–	0.85	–	-0.09
	–	(4.94)	–	(6.80)	–	(6.53)	–	(3.59)	–	(2.39)	–	(-0.26)
CIVR factor 2	–	-0.14	–	0.29	–	0.43	–	0.15	–	0.58	–	1.21
	–	(-0.62)	–	(0.97)	–	(1.30)	–	(0.68)	–	(3.04)	–	(6.71)
Adjusted R^2	0.14	0.75	0.12	0.71	0.04	0.53	0.07	0.51	0.07	0.67	0.03	0.77

Table 8: Predictive power of systematic and idiosyncratic variances.

This table reports the OLS coefficient estimates from contemporaneous and predictive regressions of macroeconomic variables on the quarterly market excess return $r_{M,t} - r_{f,t}$, market variance $\sigma^2_{M,t}$ (used as proxy for systematic variance), and common idiosyncratic variance $\overline{\sigma}^2_{\epsilon,t}$ (computed as the cross-sectional average of idiosyncratic variance). The dependent variables used in the specifications reported across columns are the quarterly market excess return, market variance, total variance (computed as the cross-sectional average of stocks' total variance), common idiosyncratic variance, GDP growth, investment growth, consumption growth, the 3-month T-bill rate, the term spread between 10-year Treasury bonds and 3-month T-bills, and the default spread between BAA and AAA corporate bonds. We conduct the regressions using both levels (Panels A and C) and innovations (Panels B and D) of the dependent and independent variables. For each series, innovations are computed as the residuals from an AR specification with the number of lags selected optimally using Schwarz' Bayesian Information Criterion (BIC). The number of lags used when computing the innovations in each series is reported in the row labeled "AR Lags (BIC)" in Panels B and D. Panels A and B report estimates from contemporaneous regressions. Panels C and D report the results of predictive regressions for a one-quarter horizon. In Panel C, in order to account for the autocorrelation of the dependent variable, we include as regressor the lagged dependent variable up to a number of lags selected optimally using Schwarz' Bayesian Information Criterion (BIC). The number of lags used is reported in the row labeled "AR Lags (BIC)" in Panel C. Newey-West t-statistics are reported in parentheses. The sample period is January 1996 to October 2009.

	Market Return$_t$	Market Variance$_t$	Total Variance$_t$	Idiosyncratic Variance$_t$	GDP Growth$_t$	Investment Growth$_t$	Consumption Growth$_t$	3-Month T-Bill$_t$	Term Spread$_t$	Default Spread$_t$
A. Contemporaneous Regression, Levels										
Market Return$_t$	–	-0.27	-0.10	-0.07	0.02	0.02	0.01	-0.02	0.02	0.01
	–	(-1.94)	(-2.10)	(-0.35)	(1.59)	(0.43)	(1.33)	(-0.86)	(1.19)	(1.28)
Market Variance$_t$	-0.73	–	1.13	0.94	-0.06	-0.28	-0.05	-0.18	0.06	0.08
	(-4.15)	–	(15.26)	(2.77)	(-3.12)	(-2.01)	(-4.04)	(-3.48)	(2.44)	(6.30)
Idiosyncratic Variance$_t$	-0.04	0.21	1.29	–	0.01	0.02	0.02	0.06	-0.02	-0.01
	(-0.35)	(3.06)	(25.17)	–	(1.35)	(0.58)	(4.62)	(3.46)	(-1.13)	(-3.87)
Observations	54	54	54	54	54	54	54	54	54	54
F-statistic	18.24	5.08	1188.31	5.54	6.71	1.94	10.99	5.63	2.02	13.86
R^2	0.27	0.41	0.99	0.27	0.41	0.24	0.42	0.27	0.08	0.73
B. Contemporaneous Regression, Innovations										
Market Return$_t$	–	-0.21	-0.07	0.00	0.03	0.12	0.01	0.01	0.01	0.00
	–	(-2.44)	(-1.24)	(-0.04)	(3.06)	(1.74)	(1.49)	(0.86)	(0.67)	(1.14)
Market Variance$_t$	-0.91	–	1.01	0.81	-0.01	0.02	-0.02	-0.02	0.02	0.05
	(-2.74)	–	(6.49)	(4.21)	(-0.52)	(0.11)	(-2.62)	(-2.74)	(1.92)	(10.97)
Idiosyncratic Variance$_t$	-0.01	0.42	1.45	–	0.00	0.02	0.01	0.01	-0.03	0.00
	(-0.04)	(2.34)	(9.68)	–	(0.20)	(0.24)	(1.85)	(1.05)	(-2.62)	(-0.41)
Observations	53	53	53	53	53	53	53	51	53	53
AR Lags (BIC)	0	1	1	1	1	1	2	4	1	2
F-statistic	8.15	3.31	258.99	12.06	5.90	1.94	6.71	4.97	2.34	82.96
R^2	0.29	0.53	0.96	0.43	0.24	0.11	0.16	0.11	0.08	0.77

continued

Table 8: Predictive power of systematic and idiosyncratic variances—continued.

	Market Return_t	Market Variance_t	Total Variance_t	Idiosyncratic Variance_t	GDP Growth_t	Investment Growth_t	Consumption Growth_t	3-Month T-Bill_t	Term Spread_t	Default Spread_t
C. Predictive Regression, Levels										
Market Return_{t-1}	0.01	-0.10	-0.48	-0.22	0.01	0.07	0.01	0.01	-0.01	-0.01
	(0.07)	(-1.27)	(-2.23)	(-1.75)	(1.08)	(1.50)	(1.05)	(2.74)	(-0.94)	(-2.16)
Market Variance_{t-1}	-0.05	0.38	-0.09	-0.40	-0.06	-0.40	0.02	0.00	0.01	-0.01
	(-0.17)	(2.35)	(-0.27)	(-2.34)	(-5.17)	(-7.59)	(0.82)	(0.14)	(0.59)	(-1.26)
Idiosyncratic Variance_{t-1}	-0.08	0.02	1.17	0.88	0.01	0.05	0.00	0.00	0.01	0.00
	(-0.78)	(0.43)	(6.41)	(8.51)	(1.29)	(1.19)	(0.04)	(0.30)	(0.71)	(-0.62)
Observations	53	53	53	53	53	53	53	51	53	53
AR Lags (BIC)	–	–	0	–	0	0	2	4	2	2
F-statistic	0.44	7.49	35.67	40.79	17.68	94.08	11.08	261.85	79.24	21.40
R^2	0.02	0.25	0.57	0.71	0.39	0.55	0.43	0.95	0.83	0.70
D. Predictive Regression, Innovations										
Market Return_{t-1}	-0.08	-0.08	-0.33	-0.16	0.00	0.05	0.01	0.01	-0.01	-0.01
	(-0.43)	(-1.18)	(-1.33)	(-1.16)	(-0.06)	(0.94)	(1.13)	(1.86)	(-1.19)	(-2.10)
Market Variance_{t-1}	-0.19	-0.20	-0.85	-0.30	-0.04	-0.31	0.02	0.01	-0.01	-0.01
	(-0.68)	(-1.04)	(-1.74)	(-1.34)	(-2.79)	(-4.18)	(1.52)	(0.97)	(-0.54)	(-0.84)
Idiosyncratic Variance_{t-1}	-0.33	0.22	0.59	0.16	0.00	0.01	-0.01	0.00	0.01	0.01
	(-1.19)	(1.84)	(1.40)	(0.70)	(0.08)	(0.06)	(-0.79)	(-0.39)	(0.44)	(1.18)
Observations	52	52	52	52	52	52	52	51	52	52
AR Lags (BIC)	0	1	1	1	1	1	2	4	1	2
F-statistic	2.40	1.41	1.33	0.75	9.66	48.09	0.83	1.18	1.35	1.51
R^2	0.09	0.06	0.07	0.05	0.15	0.37	0.06	0.07	0.06	0.08

Table 9: Campbell's (1993) ICAPM test.

This table reports the results of a vector autoregression of monthly market excess returns $r_M - r_f$, market variance σ_M^2, and cross-sectional average idiosyncratic variance $\overline{\sigma}_\epsilon^2$. t-statistics are reported in parentheses. The sample period is January 1996 to October 2009 (163 observations).

	Variable at t		
Variable at $t-1$	$r_M - r_f$	σ_M^2	$\overline{\sigma}_\epsilon^2$
Market Return $r_M - r_f$	0.14	−0.29	−0.50
	(1.67)	(−3.55)	(−4.01)
Market Variance σ_M^2	−0.04	0.67	−0.17
	(−0.63)	(10.77)	(−1.78)
Idiosyncratic Variance $\overline{\sigma}_\epsilon^2$	−0.01	0.01	0.85
	(−0.31)	(0.27)	(17.05)
χ^2-statistic	5.71	243.62	401.93
R^2	0.03	0.60	0.71

Table 10: The effect of option illiquidity and hedging costs on variance risk premia.

This table reports estimation results from Fama-MacBeth regressions of individual stocks' variance risk premia on a number of explanatory variables. The dependent variable is the total variance risk premium (left) or the idiosyncratic variance risk premium (right). Specifications (1), (2), (3) use different sets of explanatory variables. Statistically significant coefficients at the 10%, 5% and 1% level are marked with *, ** and ***, respectively. The sample period is January 1996 to October 2009.

	Total variance risk premium			Idiosyncratic variance risk premium		
	(1)	(2)	(3)	(1)	(2)	(3)
Stock bid-ask spread	-0.83	-0.57	-0.93	0.42	0.35	-0.49
Option bid-ask spread (atm)	0.00	0.01	0.01	-0.00	0.01	0.01*
Option bid-ask spread (oom)	-0.02***	-0.00	-0.00	-0.02***	0.00	0.00
Stock volume	1.05***	0.28	0.15	1.32***	0.18	0.42
Option volume > 0	1.60***	1.13***	0.53	1.62***	0.57**	0.14
Option volume	-0.35	-0.52**	-1.11***	-0.21	-0.42**	-0.81**
Open interest > 0 (atm)	-0.01	1.08	0.69	-0.25	1.18	0.30
Open interest (atm)	-0.15***	-0.07***	-0.00	-0.17***	-0.04**	-0.02
Stock skewness	–	-0.20	-0.24	–	-0.02	-0.01
Stock kurtosis	–	-0.40***	-0.33***	–	-0.61***	-0.56***
Lagged dependent variable	–	0.09***	0.11***	–	0.18***	0.20***
Realized variance	–	-0.52	-3.34	–	0.99	-2.12
β S&P	–	-1.25	-0.94	–	0.87	0.39
β Nasdaq	–	5.61***	4.76***	–	2.54***	2.81**
Market-to-book	–	–	0.20***	–	–	0.17***
Firm size	–	–	-0.08	–	–	0.04
Profitability	–	–	-3.02	–	–	-3.93**
Book leverage	–	–	-0.33**	–	–	-0.29***
Capital expenditure	–	–	1.78	–	–	0.07
Cash holding	–	–	2.15**	–	–	0.98
Dividend payer	–	–	-0.74**	–	–	-1.04***
Observations	48,166	47,330	32,265	47,982	47,060	32,093
F-statistic	6.96	9.51	4.92	14.75	45.52	18.74
R^2	0.06	0.15	0.27	0.07	0.18	0.29

Table 11: Testing the model-implied equilibrium pricing of variance risk.

This table reports the results of Fama-MacBeth regressions of individual stocks' realized variance risk premia on the following explanatory variables: their exposure to the market, size, value and momentum factors, their exposure to a factor constructed as the cross-sectional average of individual stocks' realized variance risk premium, two proxies for supply pressure arising from option compensation and covered call writing by mutual funds (as well as dummy variables capturing cases where these variables are missing), and the riskiness of each stock's "truly idiosyncratic" variance, which is computed as the time series variance of the residuals from the first pass regression of each stock's variance risk premium on the market, size, value, momentum and common variance factors. The dependent variable is the total variance risk premium (left) or the idiosyncratic variance risk premium (right). The sample period is January 1996 to October 2009.

	Total variance risk premium		Idiosyncratic variance risk premium	
	Coefficient	t-stat	Coefficient	t-stat
Constant	-3.68	(-6.69)	-0.35	(-0.87)
Market	-2.66	(-4.37)	-0.90	(-1.51)
SMB	-2.11	(-4.86)	0.12	(0.30)
HML	1.64	(4.04)	0.37	(0.97)
MOM	-2.94	(-3.71)	-1.15	(-1.50)
Common Variance	5.59	(4.20)	–	–
Common Idiosyncratic Variance	–	–	3.40	(5.01)
Supply proxies:				
Option Compensation	25.14	(5.08)	16.45	(5.30)
Option Compensation Missing	0.65	(1.16)	0.76	(1.73)
Mutual Fund Holdings	2.31	(1.10)	4.17	(2.24)
Mutual Fund Holdings Missing	-0.15	(-0.26)	0.59	(1.32)
Truly Idiosyncratic Variance Risk	2.55	(0.82)	13.40	(2.86)

Table 12: Dynamics of variance risk premia on individual stocks.

This table reports the estimates from a GARCH-M specification for the time-series behavior of the cross-sectional average variance risk premium on individual stocks. The dependent variable is the total variance risk premium (left) or the idiosyncratic variance risk premium (right). The sample period is January 1996 to October 2009.

	Total variance risk premium		Idiosyncratic variance risk premium	
	Coefficient	t-stat	Coefficient	t-stat
Mean equation				
Constant	-3.02	(-5.89)	-0.41	(-1.03)
Conditional variance	1.53	(9.01)	10.09	(5.87)
Variance equation				
Constant	0.04	(2.48)	0.00	(0.60)
ARCH (1)	0.92	(6.82)	0.23	(5.97)
GARCH (1)	0.47	(7.39)	0.81	(41.30)

Table 13: Profitability of dispersion trading strategies.

This table reports summary statistics of the monthly returns earned by a dispersion trading strategy that takes short positions in index variance swaps and long positions in individual stocks' variance swaps. The strategies in panel A have returns that are uncorrelated with variance swap returns, while those in panel B have returns that are uncorrelated with the average variance swap return of the index constituents. The sample period is January 1996 to October 2009.

Index	Mean	S.D.	Sharpe	Min	Max
A. Portfolio uncorrelated with index variance swap returns					
S&P 100	1.27	2.52	1.74	-12.79	19.21
Nasdaq 100	1.27	2.03	2.17	-11.77	13.48
B. Portfolio uncorrelated with average constituent variance swap returns					
S&P 100	1.87	3.62	1.79	-27.72	14.51
Nasdaq 100	1.69	2.93	1.99	-31.83	14.91

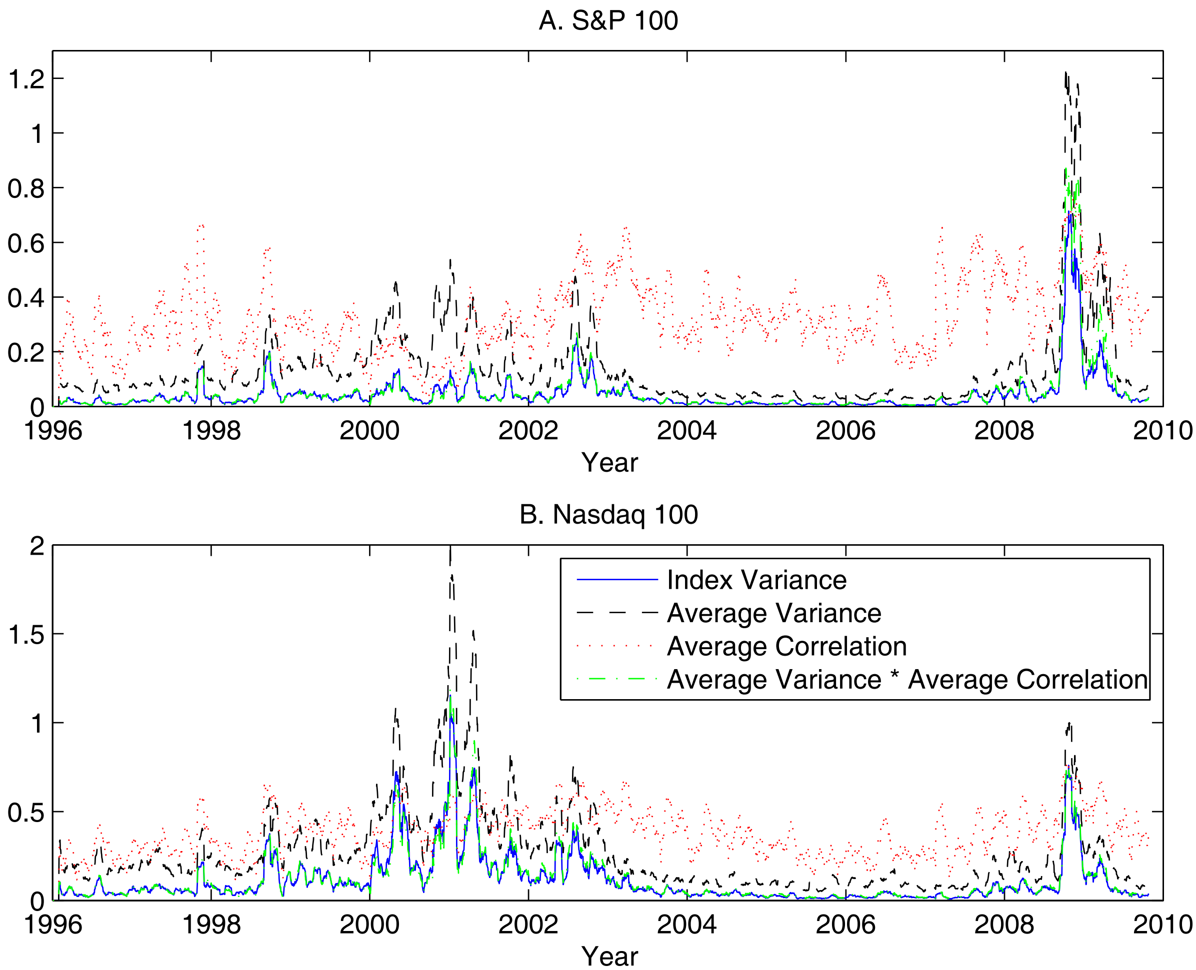

Figure 1: Relationship between index variances, constituent variances, and constituent correlations.

The figure reports realized index variance, the average realized variance of the index constituents, the average correlation between index constituents, and the product of the average realized variance and the average correlation for the S&P 100 and Nasdaq 100 indices. All series are computed using trailing 1 month (21 trading day) windows, and the averages are based on the index weights. The sample period is January 1996 to October 2009.

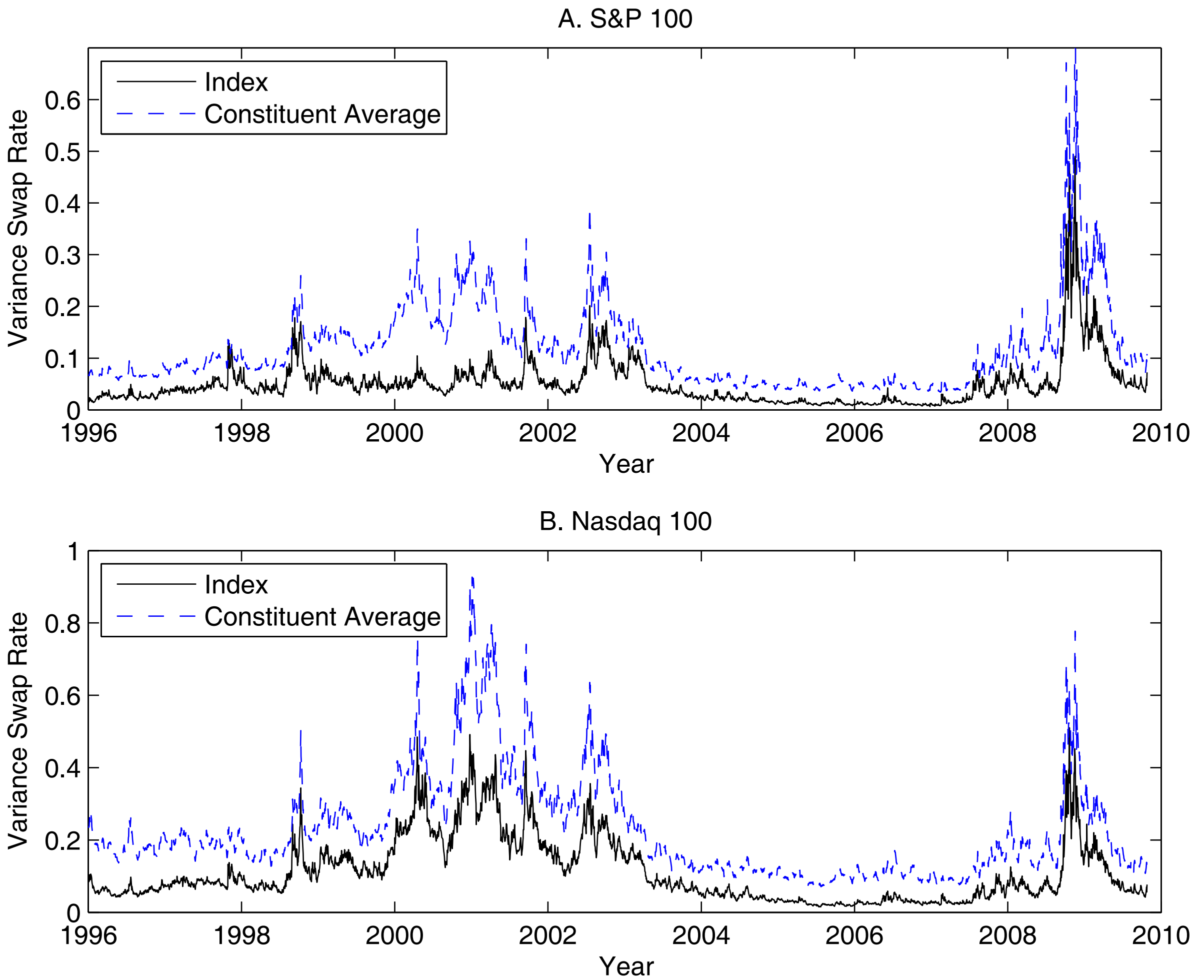

Figure 2: Relationship between index and constituent variance swap rates.

The figure shows the index variance swap rates and the average variance swap rates of the index constituents for the S&P 100 and Nasdaq 100 indices. The averages are computed using the index weights. The sample period is January 1996 to October 2009.

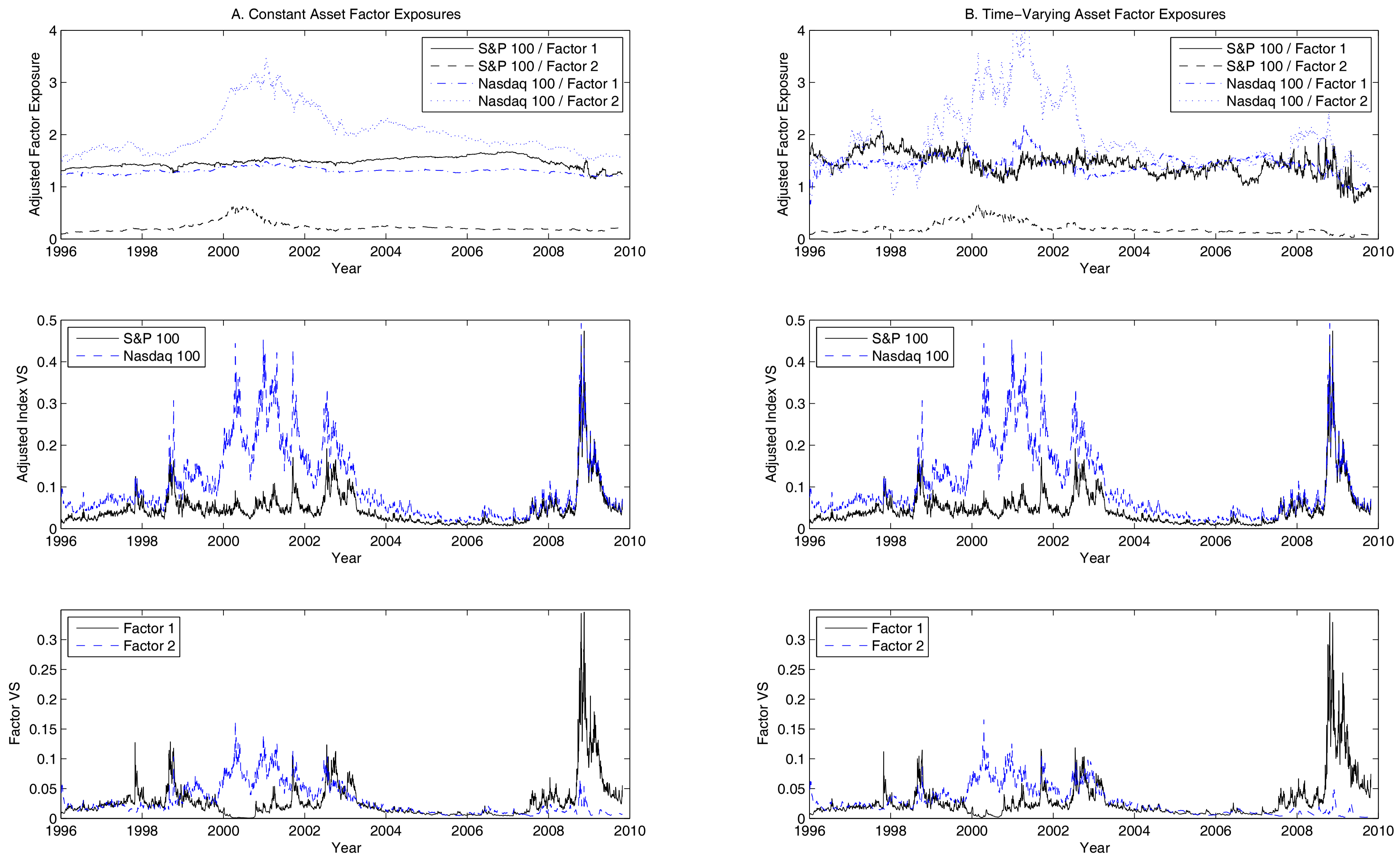

Figure 3: The time series of factor variance swap rates.

This figure illustrates the results from each step in the factor variance swap rate extraction. The left panels present the case of constant asset factor exposures, the right panels the case of time-varying factor exposures. The top panels show the adjusted factor exposures of the S&P 100 and Nasdaq 100 indices (the $A_{p,j,t}$ terms in (16)). The middle panels report the adjusted index variance swap rates (the $y_{p,t}$ terms in (15)). The bottom panels show the time series of the two factor variance swap rates estimated using the Kalman filter.