Executive Summary

Traditional mean-variance (MV) optimization (Markowitz (1952)) assumes that financial returns are normally distributed. Although as early as Kendall and Hill (1953), Mandelbrot (1963) and Fama (1965), researchers voiced concern about the existence and possible implications of skewness and kurtosis in return distributions. Especially, since the financial crisis in 2008 the discussion in asset pricing literature is drawn to the four-moment CAPM including skewness and kurtosis (see e.g. Bloechlinger (2018)). In the context of portfolio theory, there is no consensus about how to include higher moments (i.e. skewness and kurtosis) into portfolio optimization to move away from the assumption of normally distributed returns. Common methods are the estimation of the expected utility function up to the fourth moment by applying the Taylor series expansion (Jondeau and Rockinger (2006)), the use of a shortage function (Briec, Kerstens and Lesourd (2004)) and the inclusion of mean, variance, skewness and kurtosis (MVSK) into a multiple objective function – a polynomial goal programming (PGP) function – to construct an ideal portfolio (see e.g. Lai (1991)).

In this thesis, higher moments are implemented into asset allocation with the use of a PGP function. In a multivariate space besides co-variance, higher order co-moments such as co-skewness and co-kurtosis more appropriately describe dependencies between return series of financial assets (Jondeau, Jurczenko, and Rockinger (2017)). Hence, estimators of these co-moment matrices are implemented into the portfolio construction process. Existing literature does not address strategic asset allocation in the context of PGP portfolios optimized for MVSK. This thesis intends to bridge this gap in literature and analyze the PGP portfolios' properties and out-of-sample performance on the basis of two asset allocations. The first asset allocation example includes different hedge fund indices combined with a stock and

bond index. The second asset allocation is an example of a strategic asset allocation (SAA) for a broadly position investor including asset classes such as equities, bonds, private equity, hedge funds, real estate etc.

Since it is not feasible to simultaneously optimize MVSK, Lai (1991) first proposed the PGP function a two-step procedure to solve the portfolio selection with conflicting objectives. In the first step all the four moments are individually optimized. Expected return and skewness are each maximized whereas variance and kurtosis are both separately minimized under a full investment and no-short sale constraint. The resulting optimal best-case scenarios are subsequently implemented in the multi-objective function. The second step combines all the moments in one objective function by minimizing the sum of the deviations from the optimal levels determined in the first step. The investor's preferences regarding MVSK are implemented by weighting the deviations in the objective function with pre-specified preference parameters (Lai, Yu and Wang (2006)).

There is a trade-off observed between expected return, volatility, skewness and kurtosis. PGP portfolios with constrained variance to a specific level the improve skewness and kurtosis compared to the MV portfolio but sacrifice expected return. The inclusion of skewness and kurtosis in the PGP function does not lead to more diversified portfolios. The portfolios shift their allocation into different assets or adjust weights but portfolios are still concentrated in few assets.

The out-of-sample analysis of a PGP function with higher moments using total return data for two different asset allocations starting in 01.1999 until 12.2017[1] confirms that MV portfolios have a tendency to act as a skewness minimizer and kurtosis maximizer. PGP

[1] The data used in the SAA until 03.2018.

portfolios realize improved skewness and kurtosis properties. Portfolios with variance constrained to the variance of the equally-weighted portfolio, preferences only need to be specified toward expected return, skewness and kurtosis. Some of the PGP portfolios outperform the naïve MV portfolio on a risk-adjusted basis considering Sharpe and Omega ratios. Although, the outperformance highly depends on the specification of the preference parameters of the higher moments. In the hedge fund allocation portfolios with relatively low or no preference for skewness perform well, whereas in the SAA portfolios with high preference for both moments perform better, especially the realized positive skewness in the SAA seems to drive the outperformance of the PGP portfolios. No general rules as to when the PGP portfolios are more likely to outperform can be deduced. Results differ between the sets of preference parameter and the assets included in the allocation.

The inclusion of the co-skewness and co-kurtosis matrix into the optimization exacerbates estimation error due to the exponential rise of the parameters in the matrices of the third and fourth order co-moments (see e.g. Martellini and Ziemann (2010)). The drastic changes in allocations of the portfolios along the efficient frontiers as well as the drastic changes in the annually recalculated weights in the out-of-sample analysis highlight the issues with the plug-in sample estimators of the higher co-moments. In the literature about the PGP approach this issue is neglected.

Nevertheless, the analysis shows that higher moments have the potential to improve asset allocation compared to naive MV optimization. The question remains is how to properly implement them into portfolio optimization to harvest their potential. The effort of including higher moments into portfolio is deemed to be more appropriate in the case of the HF allocation than the SAA, due to higher diversity in the skewness and kurtosis values of the assets in the HF allocation.